Nonlinear dynamical analysis

Jaeseung Jeong, Ph.D
Department of Bio and Brain Engineering
KAIST
Why are brain oscillations so complex?
EEG Complexity and emotion

- Longitudinal variation of global entropy (K) and scores to mood assessment scale for each subject.
- Entropy evolution is represented with open circles, and corresponds to the left ordinate axis scale.
- Depressive mood modulation is depicted with black squares and corresponds to the right ordinate axis scale.
- Days of recording are given in abscissa. Depressed patients: Mrs. G., Mss. S. and Mrs. R.
Complexity during different sleep stages

ApEn (EEG)

Wake | REM | Stage I | Stage IV
---|---|---|---
2.0 | 2.0 | 2.0 | 1.5

+ + +

Wake > REM > Stage I > Stage IV
Approximate Entropy in AD/HD patients
This EEG time series shows the transition between interictal and ictal brain dynamics. The attractor corresponding to the interictal state is high dimensional and reflects a low level of synchronization in the underlying neuronal networks, whereas the attractor reconstructed from the ictal part on the right shows a clearly recognizable structure. (Stam, 2003)
One possible answer for why seizures occur is that:

Seizures **have to occur to reset (recover) some abnormal connections among different areas** in the brain. Seizures serve as a **dynamical resetting mechanism**.
The effect of alcohol on the EEG complexity measured by Approximate entropy
Measures of Complexity

- Most extant complexity measures can be grouped into two main categories:

- Members of the first category (algorithmic information content and logical depth) all capture the randomness, information content or description length of a system or process, with random processes possessing the highest complexity since they most resist compression.
Measures of Complexity

• The second category (including statistical complexity, physical complexity and neural complexity) conceptualizes complexity as distinct from randomness.

• Here, complex systems are those that possess a high amount of structure or information, often across multiple temporal and spatial scales. Within this category of measures, highly complex systems are positioned somewhere between systems that are highly ordered (regular) or highly disordered (random).
A schematic diagram of the shape of such measures. It should be emphasized again that a generally accepted quantitative expression linking complexity and disorder does not currently exist.
Complex Networks

- A key insight is that network topology, the graph structure of the interactions, places important constraints on the system's dynamics, by directing information flow, creating patterns of coherence between components, and by shaping the emergence of macroscopic system states.

- Complexity is highly sensitive to changes in network topology. Changes in connection patterns or strengths may thus serve as modulators of complexity.

- The link between network structure and dynamics represents one of the most promising areas of complexity research in the near future.
Why complexity?

Why does complexity exist in the first place, especially among biological systems? A definitive answer to this question remains elusive.

One perspective is based on the evolutionary demands biological systems face. The evolutionary success of biological structures and organisms depends on their ability to capture information about the environment, be it molecular or ecological.

Biological complexity may then emerge as a result of evolutionary pressure on the effective encoding of structured relationships which support differential survival.
Why complexity?

- Another clue may be found in the emerging link between complexity and network structure.
- Complexity appears very prominently in systems that combine segregated and heterogeneous components with large-scale integration.
- Such systems become more complex as they more efficiently integrate more information, that is, as they become more capable to accommodate both the existence of specialized components that generate information and the existence of structured interactions that bind these components into a coherent whole.
- Thus reconciling parts and wholes, complexity may be a necessary manifestation of a fundamental dialectic in nature (Scholapedia).
Functional segregation and integration

- While the evidence for regional specialization in the brain is overwhelming, it is clear that the information conveyed by the activity of specialized groups of neurons must be functionally integrated in order to guide adaptive behavior.

- Like functional specialization, functional integration occurs at multiple spatial and temporal scales.

- The rapid integration of information within the thalamocortical system does not occur in a particular location but rather in terms of a unified neural process.
How does the brain ‘bind' together the attributes of objects to construct a unified conscious scene?

- Neurons can integrate frequently co-occurring constellations of features by convergent connectivity. However, convergence is unlikely to be the predominant mechanism for integration.
- First, no single (‘master') brain area has been identified, the activity of which represents entire perceptual or mental states.
- Second, the vast number of possible perceptual stimuli occurring in ever changing contexts greatly exceeds the number of available neuronal groups (or even single neurons), thus causing a combinatorial explosion.
- Third, convergence does not allow for dynamic (‘on-the-fly') conjunctions in response to novel, previously unencountered stimuli.
(A) Connections between groups are arranged such that groups with similar response selectivity are preferentially connected, are arranged anisotropically along the axis of their orientation selectivity, and connection density falls off with distance. This produces spike patterns with significant correlations between some groups and not others, as well as a temporally varying EEG that reflects a mixture of synchronization and desynchronization. **Segregation and integration** are balanced and complexity is high. (B) Connection density is reduced. No statistically significant correlations exist, and a flat EEG results. (C) Connections are of the same overall density as in (A), but are spread out uniformly and randomly over the network. The system is fully integrated but functional specialization is lost, and complexity is low.
Characteristics of Complex Systems

A 'complex' system

Emergent behavior that cannot be simply inferred from the behavior of the components

Chaos
Fine Scales Influence Large Scale Behavior

Emergence
Hierarchies
Self-Organization
Control Structures
Composites
Substructure
Decomposability

A 'simple' system

Transdisciplinary Concepts
Across Types of Systems, Across Scales, and thus Across Disciplines

Involve:
Many Components
Dynamically Interacting
and giving rise to
A Number of Levels or Scales
which exhibit
Common Behaviors
EEG recordings
Several sources of complexity in EEG
Fundamental assumptions of Nonlinear dynamical analysis

- EEG signals are generated by nonlinear deterministic processes with nonlinear coupling interactions between neuronal populations.
- Nonlinear deterministic systems may show a sensitive dependence on initial conditions, implying that different states of a system, being arbitrarily close initially, can become exponentially separated in sufficiently long times. This behavior is called deterministic chaos.
- These systems behave very irregular and complex, similar to stochastic systems.
- Given the highly nonlinear nature of neuronal interactions at multiple levels of spatial scales, it is quite natural to apply nonlinear methods to the EEG.
Several limitations on Nonlinear Dynamical Analysis

The proper computations and interpretations of the nonlinear measures involves several pitfalls.

• The computation of the nonlinear measures can be biased by autocorrelation effects in the time series. This can be avoided by discarding vector pairs with time indices less than the autocorrelation time (Theiler, 1986).

• Insufficient length of the time series can bias the nonlinear measures estimate (Eckman and Ruelle, 1992).

• The computation of the nonlinear measures can be influenced by noise (Möller et al., 1989).

• [Examples] Colored, filtered noise can give rise to linear scaling regions of the plot and saturation with increasing embedding dimensions, spuriously suggesting the existence of a low-dimensional attractor (Osborne and Provenzale, 1989).
Several limitations on Nonlinear Dynamical Analysis

- The fundamental assumption of NDA that the EEG generates by a deterministic process is still disputable.
- The absolute values of nonlinear measures depend sensitively on algorithms used or parameters in the algorithms, such as the embedding dimension, the time delay, the number of data point, the cut-off noise level.
- Given that nonlinear measures like the $D_2$ or $L_1$ reflect nonlinear dynamics of the attractor in the phase space reconstructed from the EEG, the physiological implications of the changes in these measures in pathological brain states are not clear.
- Nonlinear dynamics of the EEG is possibly influenced by many physiological factors including age, sex intelligence as well as by the severity of the disease.
Is EEG deterministic or stochastic?

- If the time series are generated from deterministic systems that are governed by nonlinear ordinary differential equations, then nearby points on the phase space behave similarly under time evolution.
- These smoothness properties imply determinism.

Detecting determinism in independent components of the EEG using ICA
Detecting determinism in independent components of the EEG using ICA
Why is determinism important?

[1] Whether a time series is deterministic or not decides our approach to investigate the time series. Thus determinism test provides us with appropriate tools for analyzing EEG signals.

[2] Determinism in the EEG (or electrocorticogram) suggests that EEGs reflecting thoughts and emotion are able to be utilized in the brain-computer interface (BCI).
Three general ways to overcome these limitations

(i) Still compute classic measures, but refrain from an interpretation in terms of dimensions or deterministic chaos, and consider them as tentative indices of different brain states.

(ii) Check the validity of the results with surrogate data

(iii) Use novel nonlinear measures which attempt to characterize some of the structure of the reconstructed trajectories without making strong assumptions about the nature of the underlying dynamics.

[Examples]
Nonlinear forecasting; Unstable periodic orbits; Mutual dimension etc.
The importance of dynamical stationarity

- Time series generated from nonlinear dynamical systems exhibit nonstationary (i.e. time-dependent) based on statistical measures (weak statistics) including the mean and variance, despite that the parameters in the dynamical process all remain constant.
- It indicates that the statistical stationarity of the time series does not imply its dynamical stationarity.
- Given that the EEG is possibly generated by the dynamical, cognitive process of the brain, the dynamical nonstationarity of the EEG can reflect on the state transition of the brain.
Dynamical nonstationarity

Definition of the Dynamical stationarity:
For two consecutive windows of a non stationary dynamical system time series, there should be change in dynamic from passage of one windows to another one.

Main Hypothesis:
Since ADHD could have shorter characteristic time for attention, we could expect same order behavior inside a Cognitive State, which could be found analyzing the time criterion in loss of Dynamical Nonstationarity.
Brain Dynamics

- Experiments:
  - Microscopic (multi electrode)
  - Macroscopic (EEG or MEG)
- Mathematical modeling:
  - From spiking neuron to large scaled networks
- Comparison with experiments
- Modification of model
- System behavior: senses, learning and memory, motor behavior
Dynamics ↔ Neurobiology

- Theory driven:
  - first principles
- Reductionism:
  - simplify the world
- Qualitative understanding:
  - mathematics

- Empirical and descriptive:
  - phenomena
- Systems level:
  - details of the system
- Quantitative understanding:
  - measurements

- Paradigm Shift
  - Complex: real systems
  - Synthesis: whole-istic
  - System biology using mathematical models
  - Universality among the diversity: biology