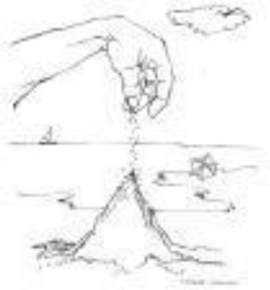
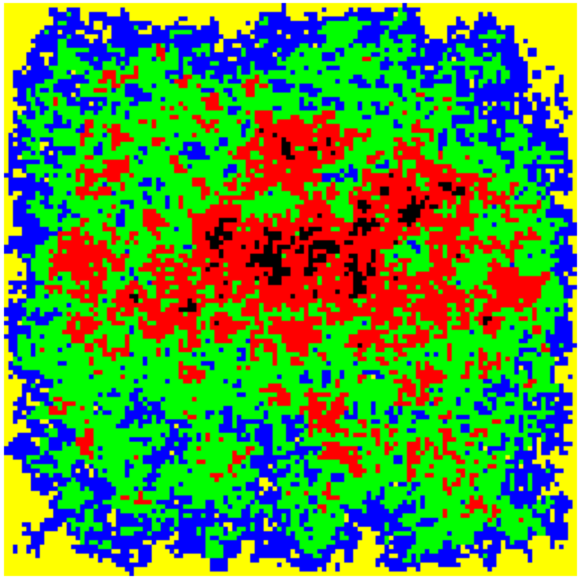


Self-organized Criticality and its implication to brain dynamics

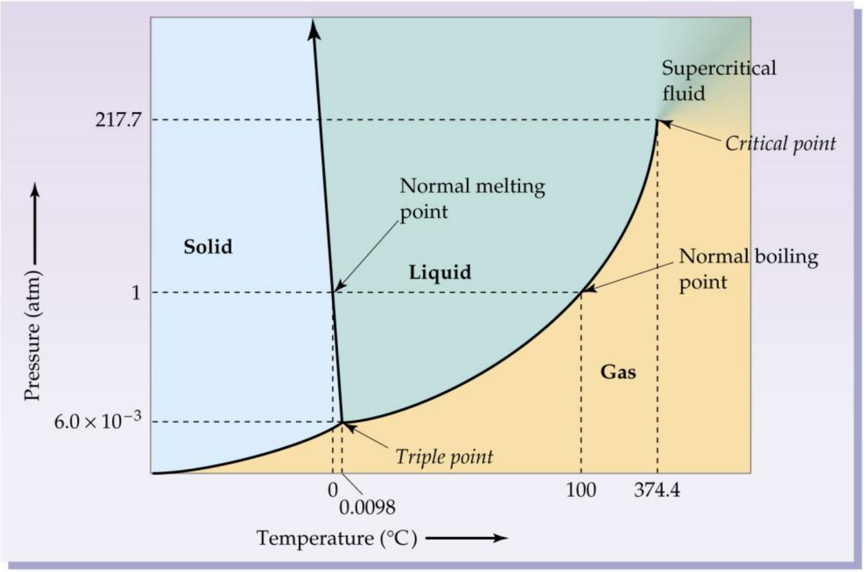


Jaeseung Jeong, Ph.D

Department of Bio and Brain Engineering,

KAIST

Criticality or critical points



- Criticality indicates the behavior of extended systems at a phase transition where scale invariance prevails.
- Many constituent microscopic parts bring about macroscopic phenomena that cannot be understood by considering a single part alone.

Self-organization

Self-organization is a ubiquitous phenomenon in biology and neuroscience including the followings:

- Spontaneous folding of proteins and other biomacromolecules
- Formation of lipid bilayer membranes,
- Homeostasis (the self-maintaining nature of systems from the cell to the whole organism)
- Flocking behavior (such as the formation of flocks by birds and schools of fish)
- The origin of life itself from self-organizing chemical systems, in the theories of autocatalytic networks

Self-organization vs. entropy

- The idea of self-organization challenges an earlier paradigm of ever-decreasing order which was based on the second law of thermodynamics in statistical mechanics where entropy is envisioned as a measure of the statistical "disorder" at a microstate level.
- However, at the microscopic level, the two need not be in contradiction: it is possible for a system to reduce its entropy by transferring it to its environment.
- Since isolated systems cannot decrease their entropy, only open systems can exhibit self-organization. However, such a system can gain *macroscopic* order while increasing its overall entropy. Specifically, a few of the system's macroscopic degrees of freedom can become more ordered at the expense of microscopic disorder.

Sandpile criticality

- Like a child's sand castle, seeds rain down onto a mound that grows until it can't grow any more. A complex balancing act keeps the mound stable until it reaches the "point of criticality"-the point at which the system "fails," creating avalanches that change the mound's shape.
- Each avalanche relieves pressure on the pile, allowing it to begin growing again. Similar patterns of alternating growth and failure show up in many complex systems.

Getting on top of self-organized criticality

Take a sand pile built up by dropping grains onto it at a steady rate. The angle of the slope will be controlled by the frictional strength of the material. This was well known to Coulomb in the eighteenth century.

Sandpile problem

- If the slope gets too steep as more sand grains are added an avalanche of grains will occur. The angle of the slope will be maintained at some critical angle by avalanches: this is self-organized criticality.
- Adding a grain of sand to the pile may trigger an avalanche of almost any size, large or small, and the distribution of these sizes will follow a power law that contains many small avalanches and just a few large ones.
- As a power-law distribution is the only distribution whose mathematical form contains no references to scale, so the concept of self-organized criticality is intimately tied to the idea of scale invariance.
- Moreover, even when sand grains are dropped onto the top of a sand pile at a steady rate, the avalanches will not occur at regular intervals in time or space, which introduces inherent unpredictability into the system

Sandpile problem

- This system is interesting in that it is attracted to its critical state, at which point the correlation length of the system and the correlation time of the system go to infinity, without any fine tuning of a system parameter.
- This contrasts with earlier examples of critical phenomena, such as the phase transitions between solid and liquid, or liquid and gas, where the critical point can only be reached by precise tuning (usually of temperature).
- Hence, in the sandpile model we can say that the criticality is self-organized.

Self-organized criticality of tropical rainfall

Both tropical rainfall and magnetism are described by the mathematics of "self-organized criticality."

Self-organized criticality of tropical rainfall

- Statistical physicists have gained a deeper insight into rainfall patterns and atmospheric dynamics by using techniques originally developed for magnetic materials.
- the onset of intense tropical rain can be described by the same mathematics as a piece of iron that's making the transition from unmagnetized to magnetized.
- To illustrate the principle, imagine that you're dribbling rice grains onto a steadily accumulating mound of rice. Adding one more grain usually does nothing at all, except to make the pile a bit bigger.
- But eventually, as the sides of the pile get steeper, the balance becomes so precarious that a single falling grain can trigger an avalanche -- sometimes even a catastrophic avalanche. At this point of "self-organized criticality," a tiny perturbation can produce a huge response.

Self-organized criticality of tropical rainfall

- It is with rainfall over the tropical oceans based on satellite remote sensing data. The atmosphere has a tendency to move to a critical point in water vapor where the likelihood of rain dramatically increases.
- The system reaches a point where it's just about to rain; it's highly susceptible. Any additional water vapor can produce a large response.
- It's known that as water vapor increases, there should be an onset of precipitation. These results tell us much more precisely how that transition occurs--which we can incorporate into atmospheric models".

Definition of self-organized criticality

- In physics, is a property of (classes of) dynamical systems which have a critical point as an attractor. Their macroscopic behavior displays the spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to precise values.
- The phenomenon was first identified by Per Bak, Chao Tang and Kurt Wiesenfeld ("BTW") in a seminal paper published in 1987 in *Physical Review Letters*, and is considered to be one of the mechanisms by which complexity arises in nature. Its concepts have been enthusiastically applied across fields as diverse as geophysics, physical cosmology, evolutionary biology and ecology, economics, quantum gravity, sociology, solar physics, plasma physics and others.
- SOC is typically observed in slowly-driven non-equilibrium systems with extended degrees of freedom and a high level of nonlinearity. Many individual examples have been identified since BTW's original paper, but to date there is no known set of general characteristics that *guarantee* a system will display SOC.

Self-organized criticality: An explanation of the 1/*f* noise

Per Bak, Chao Tang, and Kurt Wiesenfeld

Physics Department, Brookhaven National Laboratory,

Upton, New York 11973

They show that dynamical systems with spatial degrees of freedom naturally evolve into a self-organized critical point. Flicker noise, or *1/f* noise, can be identified with the dynamics of the critical state. This picture also yields insight into the origin of fractal objects.

Bak, Tang and Wiesenfeld's 1987 paper

- They linked together these factors: a simple cellular automaton was shown to produce several characteristic features observed in natural complexity (fractal geometry, 1/f noise and power laws) in a way that could be linked to critical-point phenomena.
- Crucially, however, the paper demonstrated that the complexity observed emerged in a robust manner that did not depend on finely-tuned details of the system: variable parameters in the model could be changed widely without affecting the emergence of critical behaviour (hence, *self-organized* criticality).
- Thus, the key result of BTW's paper was its discovery of a mechanism by which the emergence of complexity from simple local interactions could be *spontaneous* — and therefore plausible as a source of natural complexity — rather than something that was only possible in the lab where it was possible to tune control parameters to precise values.

Universality of self-organized criticality

- Despite a fundamental drift to increasing disorder, we observe order at every scale in the universe from super clusters of galaxies to the hexagonal symmetry of snowflakes. This order has culminated with the human brain.
- Recent advances in our understanding of the ability of multi-component systems to apparently self-organize into ordered structures in which characteristic length and time scales disappear, so called Self-organized criticality (SOC), may explain the origin of spatial and temporal fractals which are common in nature.
- They may also represent a first step in explaining the fundamental transition between a chemical soup governed by the laws of physics, to complex self-reproducing molecules whose further development was governed by the laws of natural selection.

Universality of self-organized criticality

- Self-organized criticality is one of a number of important discoveries made in statistical physics and related fields over the latter half of the 20th century, discoveries which relate particularly to the study of complexity in nature.
- For example, the study of cellular automata, from the early discoveries of Stanislaw Ulam and John von Neumann through to John Conway's Game of Life and the extensive work of Stephen Wolfram, made it clear that complexity could be generated as an emergent feature of extended systems with simple local interactions.
- Over a similar period of time, Benoît Mandelbrot's large body of work on fractals showed that much complexity in nature could be described by certain ubiquitous mathematical laws, while the extensive study of phase transitions carried out in the 1960s and '70s showed how scale invariant phenomena such as fractals and power laws emerged at the critical point between phases.

Some definitions of SOC

- Self-organized criticality (SOC) is a *concept* to describe emergent complex behavior in physical systems (Boettcher and Percus 2001)

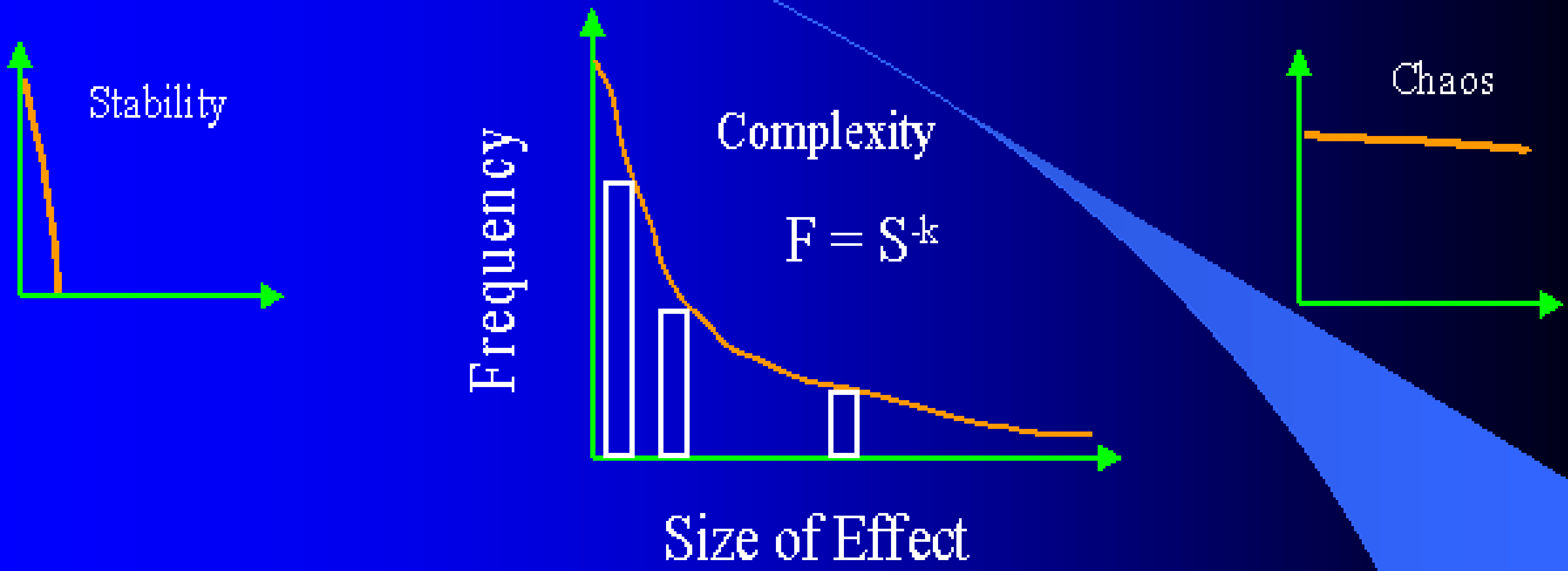
- SOC is a *mechanism* that refers to a dynamical process whereby a non-equilibrium system starts in a state with uncorrelated behavior and ends up in a complex state with a high degree of correlation (Paczuski et al. 1996)

The HOW and WHY of SOC are not generally understood

Self-Organized Criticality (Per Bak, 1988)

- State of a system in which the effects of a **constant-size** perturbation vary from small to large according to a power law:
  - N = # effects of a given size; S = size of effect
  - $N = S^{-k}$ or $\ln N = -k*\ln S$ (i.e. Linear log-log plot)
- Edge of Chaos ("Complexity") = Critical state
  - Simple perturbations will normally have small effects, but occasionally large ones when they push the system into chaos
- Fossil records of extinction events
  - Show a power-law distribution => biosphere in critical state
  - Major extinction events (loss of many species) need not require catastrophe theory as an explanation. The chance small perturbation may be enough.

Power Law Distributions



- All effects are triggered by the same type of disturbance.
- Hallmark of the Complexity (the edge of chaos).
- Homeostasis: General stability in face of perturbations
- Adaptation: Occasional large changes.
- Catastrophies: Big effects can result from small perturbations

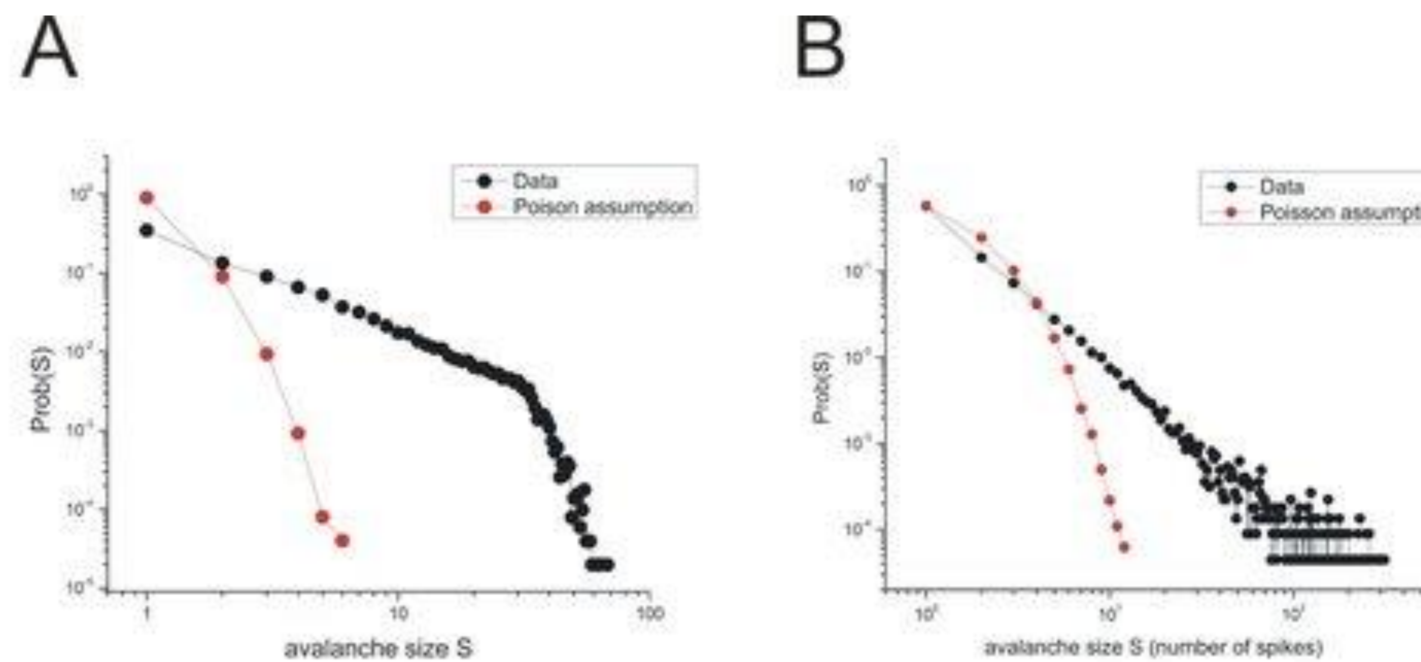
SOC and 1/f noise

- Once the sandpile model reaches its critical state there is no correlation between the system's response to a perturbation and the details of a perturbation.
- Generally this means that dropping another grain of sand onto the pile may cause nothing to happen, or it may cause the entire pile to collapse in a massive slide. The model also displays 1/f noise, a feature common to many complex systems in nature.

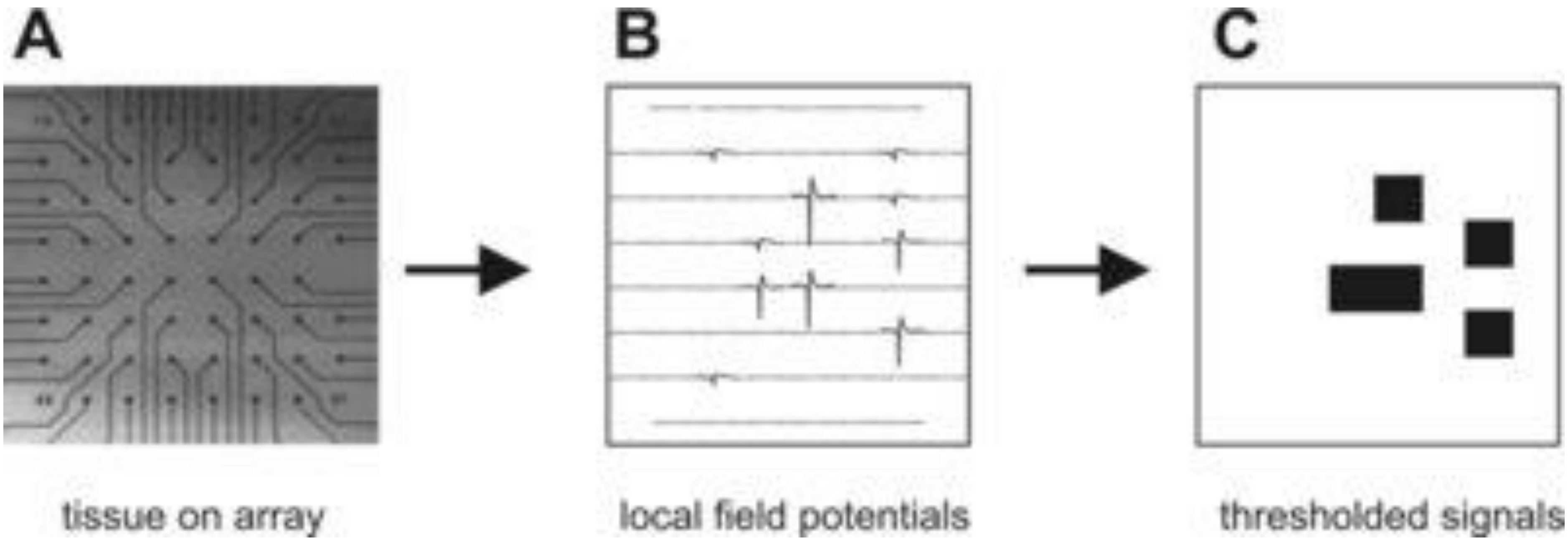
Important issues on SOC

- Due to BTW's metaphorical visualization of their model as a "sandpile" on which new sand grains were being slowly sprinkled to cause "avalanches", much of the initial experimental work tended to focus on examining real avalanches in granular matter, the most famous and extensive such study probably being the Oslo ricepile experiment.
- Other experiments include those carried out on magnetic-domain patterns, the Barkhausen effect and vortices in superconductors. Early theoretical work included the development of a variety of alternative SOC-generating dynamics distinct from the BTW model, attempts to prove model properties analytically, and examination of the necessary conditions for SOC to emerge.
- One of the important issues for the latter investigation was whether conservation of energy was required in the local dynamical exchanges of models: the answer in general is no, but with reservations, as some exchange dynamics do require local conservation at least on average. In the long term, key theoretical issues yet to be resolved include the calculation of the possible universality classes of SOC behavior and the question of whether it is possible to derive a general rule for determining if an arbitrary algorithm displays SOC.

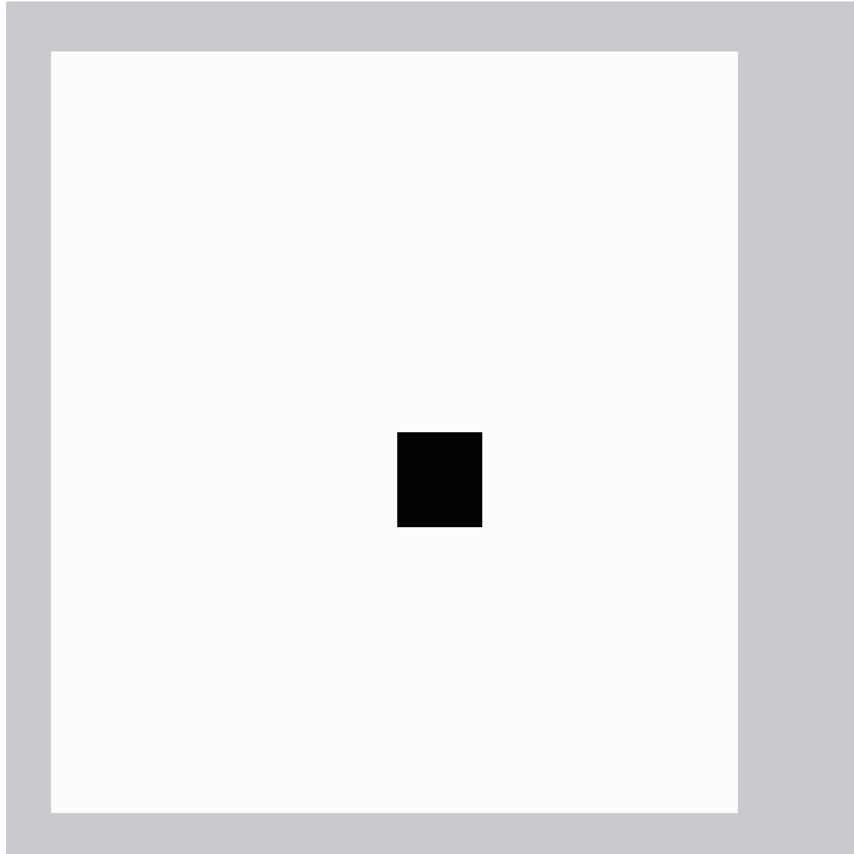
Neuronal avalanche



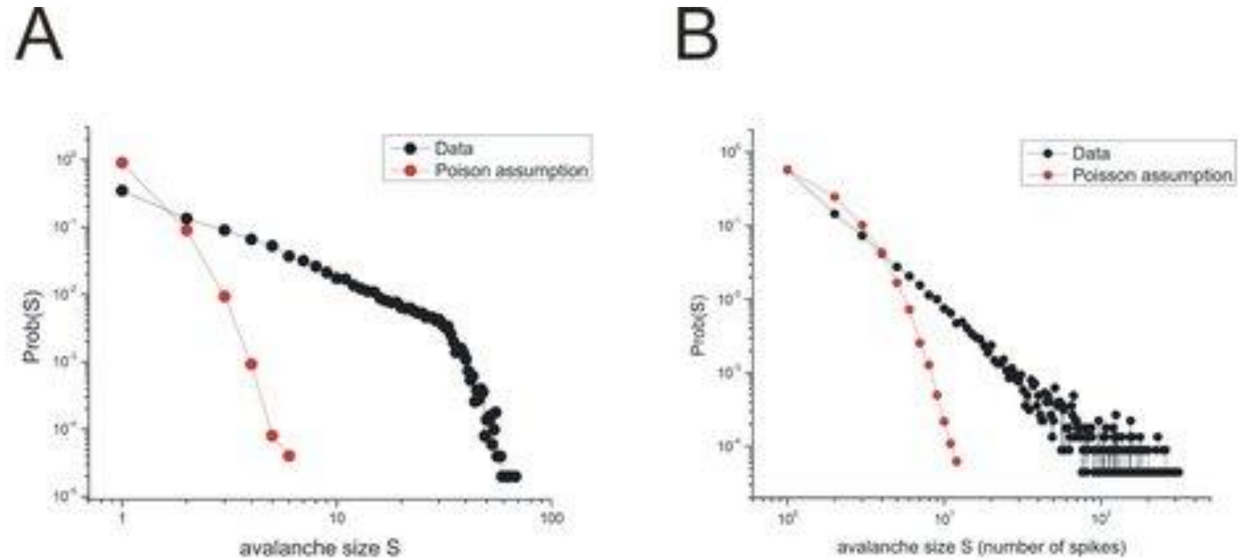
- A **neuronal avalanche** is a cascade of bursts of activity in neuronal networks whose size distribution can be approximated by a power law, as in critical sandpile models (Bak et al. 1987).



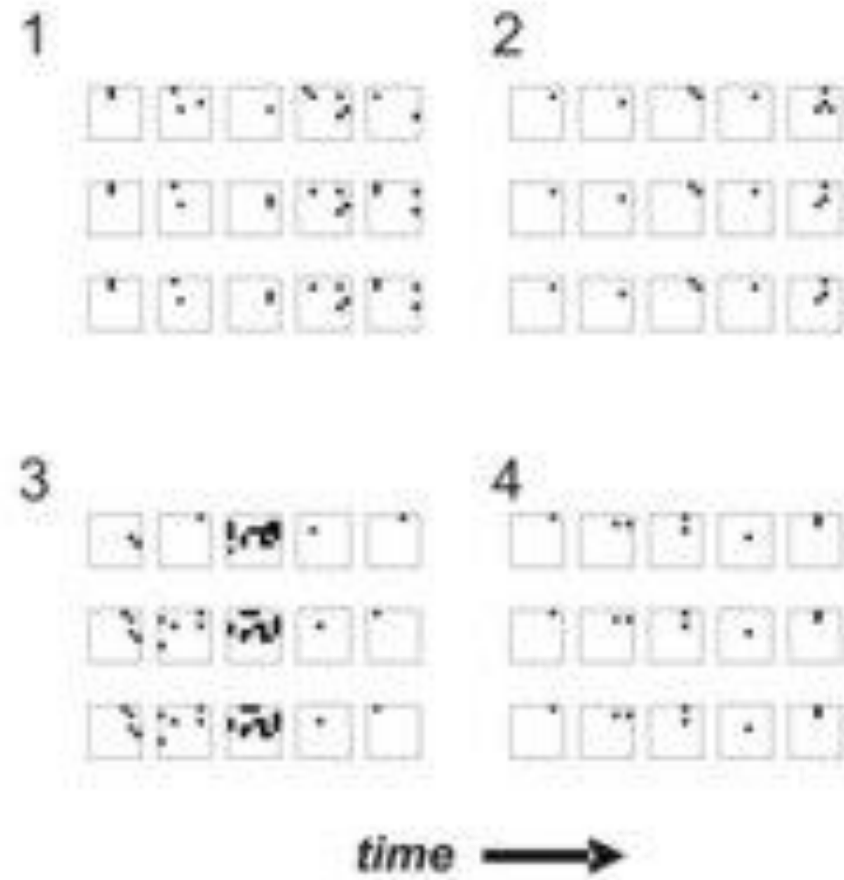
- Neuronal avalanches are seen in cultured and acute cortical slices (Beggs and Plenz, 2003; 2004). Activity in these slices of neocortex is characterized by brief bursts lasting tens of milliseconds, separated by periods of quiescence lasting several seconds. (Local field potentials (LFPs) that exceed three standard deviations are represented by black squares.)



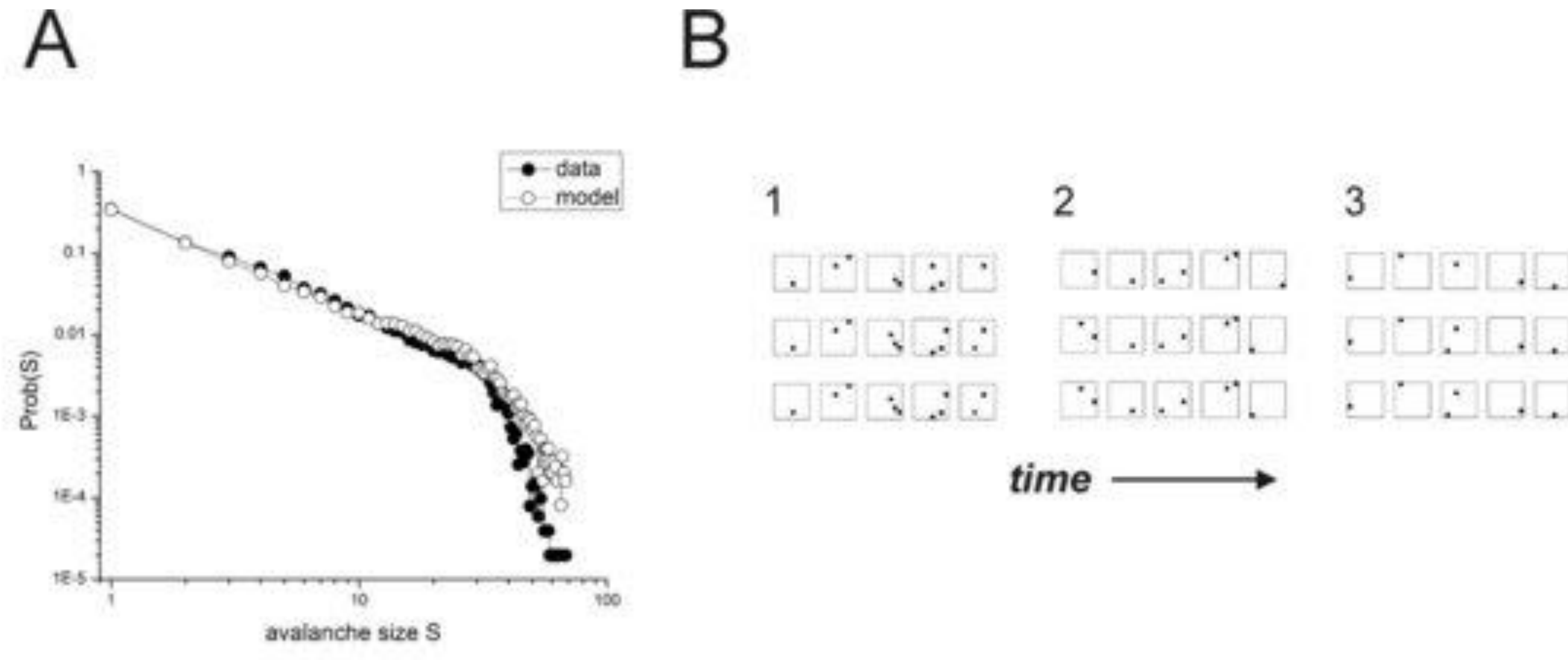
- When observed with a multielectrode array, the number of electrodes driven over threshold during a burst is distributed approximately like a power law. Although this phenomenon is highly robust and reproducible, its relation to physiological processes in the intact brain is currently not known. (Neuronal avalanches in an acute cortical slice.)



- Avalanche size distributions. A, Distribution of sizes from acute slice LFPs recorded with a 60 electrode array, plotted in log-log space. The straight line is indicative of a power law, suggesting that the network is operating near the critical point B, Avalanche size distribution for spikes can be approximated by a straight line over three orders of magnitude in probability, without a sharp cutoff as seen in panel A. Data were collected with a 512 electrode array from an acute cortical slice bathed in high potassium and zero magnesium Data were binned at 1.2 ms to match the short interelectrode distance of 60 µm.



- Families of repeating avalanches from an acute slice. Each family (1-4) shows a group of three similar avalanches. Repeating avalanches also occur in cortical slice cultures. Repeating avalanches are stable for 10 hrs and have a temporal precision of 4 ms, suggesting that they could serve as a substrate for storing information in neural networks.



- A branching model captures the two main features of the data. Model was tuned to the critical point such that the branching parameter, σ, equaled unity.

Implications of neuronal avalanche

- *Information transmission*.

When neural networks are tuned to the critical point, they have optimal information transmission (Beggs and Plenz, 2003; Bertschinger and Natschlager, 2004; Kinouchi and Copelli, 2006), because there is a balance between strong signal propagation and resistance to saturation.

- *Information storage*.

When a recurrent network based on a branching process is tuned to the critical point, the number of significantly repeating avalanche patterns is maximized (Haldeman and Beggs, 2005). At the critical point, there is a mixture of strong and weak connections, allowing for a variety of independently stable patterns of activity.