A Method for Determinism in Short Time Series, and its Application to Stationary EEG

Jaeseung Jeong*, John C. Gore, and Bradley S. Peterson

***Abstract*—A novel method for detecting determinism in short time series is developed and applied to investigate determinism in stationary electroencephalogram (EEG) recordings. This method is based on the observation that the trajectory of a time series generated from a differentiable dynamical system behaves smoothly in an embedded state space. The angles between two successive tangent vectors in the trajectory reconstructed from the time series is calculated as a function of time. The irregularity of the angle variations obtained from the time series is estimated using second-order difference plots, and compared with that of the corresponding surrogate data. Using this method, we demonstrate that scalp EEG recordings from normal subjects do not exhibit a low-dimensional deterministic structure. This method can be useful for analyzing determinism in short time series, such as those from physiological recordings.**

***Index Terms*—Determinism, smoothness, stationary EEG, time series.**

I. Introduction

An important problem in the study of a periodic and apparently irregular time series is determining whether the time series arises from a stochastic process or has deterministic component that is generated from chaotic dynamics having finite degrees of freedom. Whether a time series has a deterministic component or not in turn dictates what approaches are appropriate for investigating the time series and its generating system.

Several methods of nonlinear dynamical analysis have previously been developed to detect determinism in time series [1]–[5]. These methods are all based on the assumption that a trajectory in the state space reconstructed from a deterministic time series behaves similarly to nearby trajectories as time evolves. Thus, a large number of data points are required to have sufficient information on nearby trajectories to compare their future behaviors. In addition, the application of these methods can lead to spurious results, if the time series under study is nonstationary. Furthermore, acquiring this large number of data points from a stationary time series is almost impossible when working with real biological systems.

The aim of this study is to develop a method for detecting determinism in short time series, and then apply it to stationary segments of an electroencephalogram (EEG) record. The method is based on the observation that successive tangent vectors in the trajectory of an attractor evolve smoothly in state space when the time series from which it has been reconstructed has been generated by a differentiable dynamical system. The validity of the assumption that is central to our approach—the first differentiability of the attractor's trajectory is useful in the detection of determinism—has been mathematically proved using a measure-based approach by Ortega and Louis [6]. It has been shown that statistical differentiability of the natural invariant measure along the reconstructed trajectory implies smoothness or determinism in a time series. An advantage of this method is no requirement of an excessively large number of data points, because the entire attractor is not needed to be constructed in order to measure the variability of its component trajectories.

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II. Methods

A. Algorithm for Detecting Determinism

Let an observed time series $x(t)$ be the output of a differentiable dynamical system f^t on an m-dimensional manifold M. First, a one-dimensional time $x(t)$ series is transformed into a multidimensional state space. To unfold the projection back onto a multivariate state space that is a representation of the original system, we use the delay coordinates [7]

$$\vec{X}(t) = [x(t),\, x(t+T),\, \ldots,\, x(t+(d-1)T)] \tag{1}$$

from a single time series $x(t)$ after performing an embedding procedure. $X(t)$ is one point of the trajectory in the state space at time t, $x(t+iT)$ are the coordinates in the state space corresponding to the time-delayed values of the time series, T is the time delay between the points of the time series considered, and d is the embedding dimension.

Our test for determinism in a time series is predicted upon assessing the smoothness of a trajectory in state space of the attractor that has been reconstructed from the time series. The trajectory of its attractor should evolve smoothly because the differentiability of the original system is preserved in state space. Thus, the angles between successive tangent vectors that are tangential to the trajectory should vary slowly, whereas the directions of tangent vectors reconstructed from stochastic time series should be random, even if they are auto-correlated.

Next, the angles between successive tangent vectors are iteratively measured along the trajectory in state space. We define the tangent vector of a trajectory in the state space as follows:

$$\vec{Y}(t) = \vec{X}(t+1) - \vec{X}(t). \tag{2}$$

Then the angles between successive tangent vectors are computed by

$$R(t) = \frac{\vec{Y}(t+1)\cdot\vec{Y}(t)}{\left|\vec{Y}(t+1)\right|\left|\vec{Y}(t)\right|}. \tag{3}$$

$R(t)$ is the cosine function of the angle between successive tangent vectors. An R value of unity indicates that the successive tangent vectors $Y(t+1)$ and $Y(t)$ are parallel. The value of R decreases as the angle between successive tangent vectors increases. It was verified that $R(t)$ was not more sensitive to the presence of noise than angles themselves.

Although the absolute value of the angles between tangent vectors depends on the correlation of the time series, the regularity of the angles may reflect smoothness, and consequently determinism of the time series [6]. Fig. 1(a) and (b) demonstrates, for example, that the $R(t)$ between successive tangent vectors for the Lorenz time series [$\dot{x} = 10(y-x)$, $\dot{y} = 28x - y - xz$, $\dot{z} = -(8/3)z + xy$ $x = 10(y-x)$] (a sampling time $t = 0.01$, the number of data points of 2000, an embedding dimension of seven, and a time delay of 15) behaves more smoothly than that for stochastic time series having an identical power spectrum [8].

Next, a second-order difference plot and a central tendency measure are used to quantify the irregularity, or angle variations, of the suc-

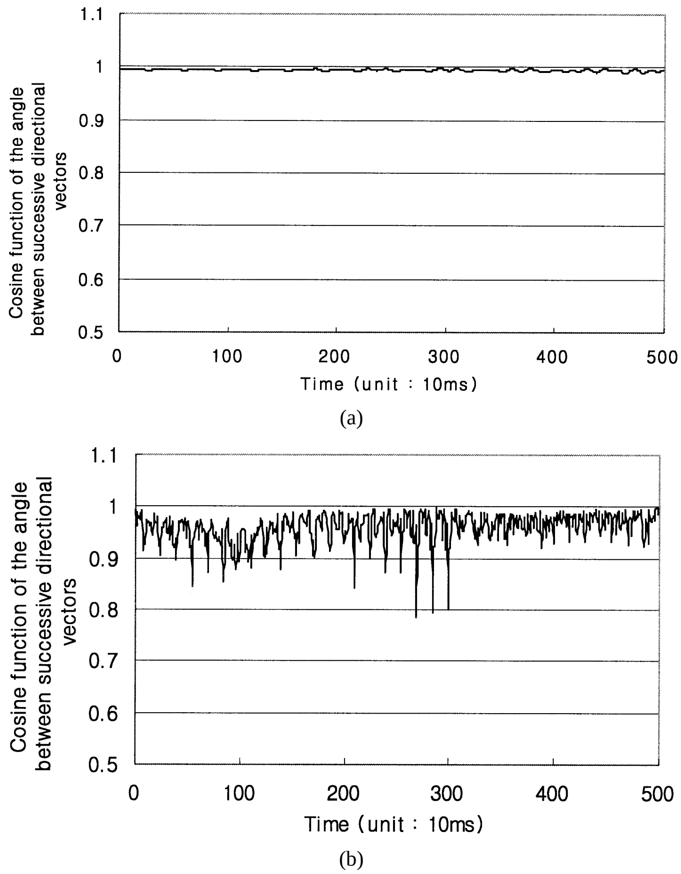

Fig. 1. The angle series of the successive tangent vectors of the trajectory in the state space reconstructed from (a) the Lorenz data (2000 data points) and (b) those from their surrogate data as a function of time with an embedding dimension of seven and a time delay of 15.

cessive tangent vectors along the trajectory. The second-order difference plot (SODP) is a graphical representation of the rate of variability: $(R_{n+2} - R_{n+1})$ versus $(R_{n+1} - R_n)$, as proposed by Cohen *et al.* [9]. The central tendency measure (CTM) provides a rapid quantitative estimate for the variability in the SODP. The CTM is computed by the average length of the points from the origin in the SODP

$$\mathrm{CTM} = \frac{1}{N-2} \sum_{n=1}^{N-2} \sqrt{(R_{n+2} - R_{n+1})^2 + (R_{n+1} - R_n)^2} \quad (4)$$

where N is the total number of points. The larger CTM of the angle variations, the less smooth is the attractor's trajectory. The index of smoothness of the trajectory S is defined as a ratio of the CTM of the angle variations of the successive tangent vectors for the original time series to that for its surrogate data. It is an effective measure for smoothness of the trajectory and, thus, for determinism in the time series.

The method of surrogate data is used to help detect nonlinear determinism. The surrogate data are linear stochastic time series that have the same power spectra as the raw time series. They are randomized to ensure the absence of any deterministic nonlinear structure. We use "iteratively refined surrogate data," which have the same autocorrelation function, Fourier power spectrum, and probability distribution as the original time series [8]. The end-point mismatch measure d_{jump} and the mismatch in the first derivative are also used to minimize spurious high-frequency component due to a discontinuity between the beginning and end of the segment. More detailed algorithms used in this study are present in the paper of Schreiber and Schmitz [8].

Fig. 2(a) and (b) presents the SODPs of the angles between successive tangent vectors for the Lorenz data and its surrogate data. The

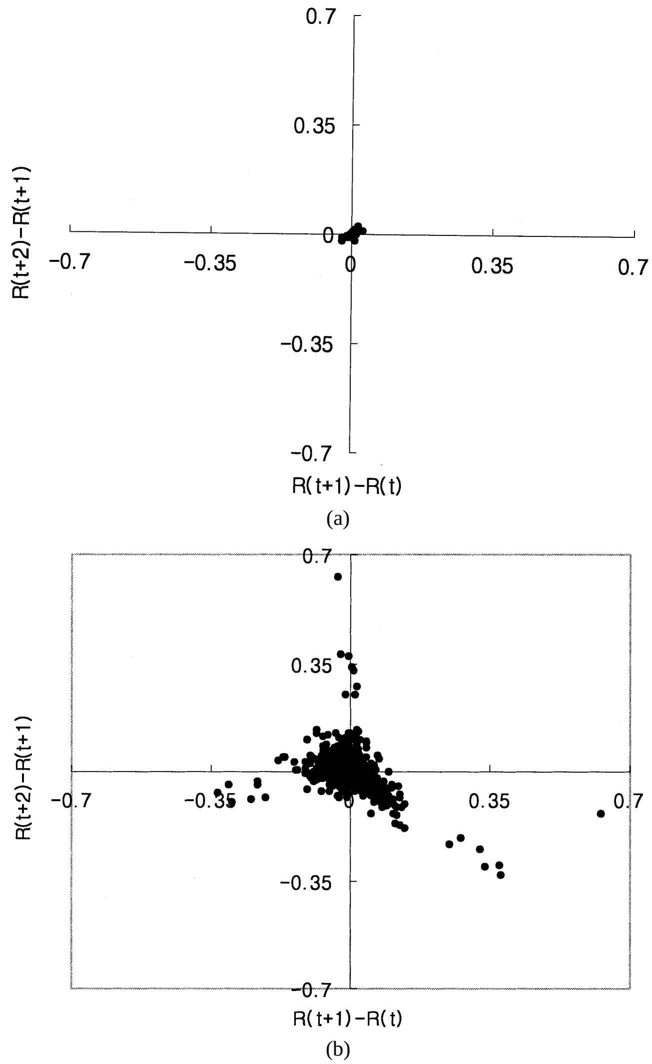

Fig. 2. Second-order difference plots of the angle series from successive tangent vectors of the trajectory in (a) the Lorenz data and (b) their surrogate data.

SODP for the Lorenz data with low variability have points clustered around the origin, whereas the SODP for their surrogate data have a much large distribution of points, indicating high variability of successive tangent vectors. The CTM values for the Lorenz data and their surrogate data in this figure are 0.002 and 0.035, respectively. We generate 20 surrogate data from the Lorenz data to test the stability of the CTM values of the surrogate data. Student t-tests show that the CTM value for the Lorenz data is significantly different from mean CTM values for its surrogate data sets (Table I). The mean of S for the Lorenz data is 0.057 ± 0.002, indicating that the Lorenz data are very smooth ($p < 0.0001$).

This method is applied to the Rössler data and Van der Pol data as well. The Rössler time series is derived from the x-component of Rössler equations $[\dot{x} = -z - y,\ \dot{y} = x + 0.15y,\ \dot{z} = 0.2 + z(x - 10)]$ with sampling time $t = \pi/100$. The SODP for 2000 Rössler data points is estimated using an embedding dimension of seven and a time delay of 44. The Van der Pol time series is taken from the x-component of Van der Pol equations $[\dot{x} = y,\ \dot{y} = 5y(1 - x^2) - 5x,\ \dot{z} = 1]$ with sampling time $\Delta t = 1/100$. The CTM for each time series differs significantly from the CTMs of their corresponding surrogate data (Table I), indicating that the attractors reconstructed from the original time series have much smoother trajectories in state space than do the time series reconstructed from their surrogate data.

TABLE I
THE COMPARISON OF THE CENTRAL TENDENCY MEASURES OF THE ORIGINAL TIME SERIES (CTM_{org}) FROM DIFFERENT DYNAMICAL SYSTEMS AND THEIR SURROGATE DATA (CTM_{sur})

Simulated data	d, T	CTM_{org}	CTM_{sur}	S score	P
Lorenz data	7, 15	0.002	0.034±0.003	0.057±0.002	<0.0001
Rossler data	7, 44	0.001	0.35±0.09	0.030±0.008	<0.0001
Van der Pol data	7, 15	0.0002	0.68±0.08	0.0003±0.0001	<0.0001
High-dimensional signal	10, 20	0.0088	0.54±0.11	0.016±0.011	<0.0001
1/f noise	10, 20	0.119	0.133±0.068	0.895±0.054	NS

TABLE II
THE COMPARISON OF THE CENTRAL TENDENCY MEASURES OF THE LORENZ DATA (CTM_{org}) IN THE PRESENCE OF VARIOUS LEVELS OF WHITE NOISE AND THEIR SURROGATE DATA (CTM_{sur}) WITH $d = 5$ AND $T = 10$

Data	CTM_{org}	CTM_{sur}	S score	P
Noise-free Lorenz data	0.002	0.034±0.003	0.059±0.001	<0.0001
Lorenz data + 10% white noise	0.019	0.071±0.015	0.267±0.004	<0.001
Lorenz data + 50% white noise	0.078	0.139±0.016	0.561±0.023	<0.01
Lorenz data + 100% white noise	0.146	0.213±0.045	0.685±0.065	<0.01
Lorenz data + 200% white noise	0.243	0.269±0.043	0.903±0.071	NS

To examine the applicability of this method to high-dimensional systems, the smoothness of the high-dimensional time series is estimated [10]. The data are generated from nonlinear coupled equations ($\dot{x}_1 = x_2$, $\dot{x}_2 = ((x_5 - 25)/3)\sin 30t + 3x_7 \sin 65t + x_{11}\sin 80t - 3|x_6|x_2 - x_9x_1$) of 12 variables including the Lorenz equation ($\dot{x}_3 = 10(x_4 - x_3)$, $\dot{x}_4 = -x_3x_5 + 28x_3 - x_4$, $\dot{x}_5 = x_3x_4 - (8/3)x_5$), the Ueda equation ($\dot{x}_6 = x_7$, $\dot{x}_7 = -0.1x_7 - x_6^3 + 12\cos 1t$), the two-well potential Duffing–Homes equation ($\dot{x}_8 = x_9$, $\dot{x}_9 = 0.15x_9 + 0.5x_8(1 - x_8^2) + 0.15\cos 0.8t$) and the Rössler equation ($\dot{x}_{10} = -(x_{11} + x_{12})$, $\dot{x}_{11} = x_{10} + 0.15x_{11}$, $\dot{x}_{12} = 0.15 + x_{12}(x_{10} - 10)$). The high-dimensional signal is found to have smoother trajectories than its surrogate data (Table I), demonstrating the relevance of this method for the assessment of determinism in high-dimensional dynamical systems.

In addition, the ability of the method to avoid the false positive designation of determinism for auto-correlated noise is tested. We use a sequence of numbers that is generated by tossing imaginary dice successively and that produces $1/f$ noise, as shown by Gardner [11]. The detailed algorithm is demonstrated in Jeong *et al.* [12]. The resulting sequence of sums is auto-correlated, with a power spectrum having a $1/f$ distribution at the range of about 10–30 Hz. Both the original and surrogate time series exhibit a large variability in their SODPs. The surrogate data have a somewhat higher SODP variability, but the CTMs for the $1/f$ noise and its surrogate data do not differ significantly (Table I).

Furthermore, the applicability of this method to deterministic systems in the presence of noise is investigated, because most experimental data from physiological systems are corrupted by noise. As the level of white noise added increased up to 100%, significant differences of CTMs between the Lorenz data and their surrogate data are detected but decreased. However, no significant difference of the CTMs is found in the presence of 200% white noise (Table II). It suggests that this method should be used with much caution to examine noisy real sys-

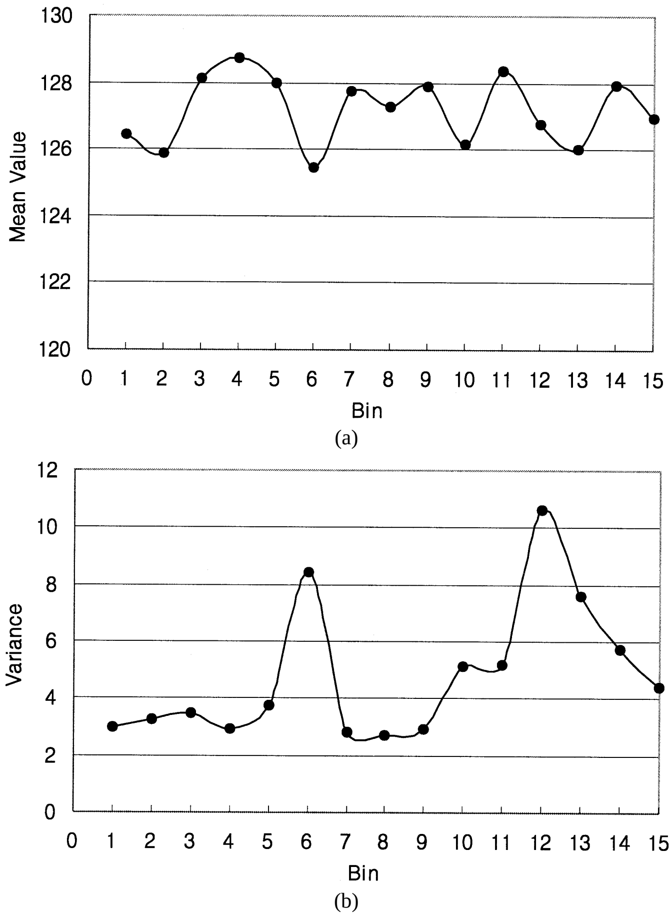

Fig. 3. Stationarity of the EEG. (a) The mean and (b) variance value for bins (bin size: 1s) for an EEG record at F_3 from a subject.

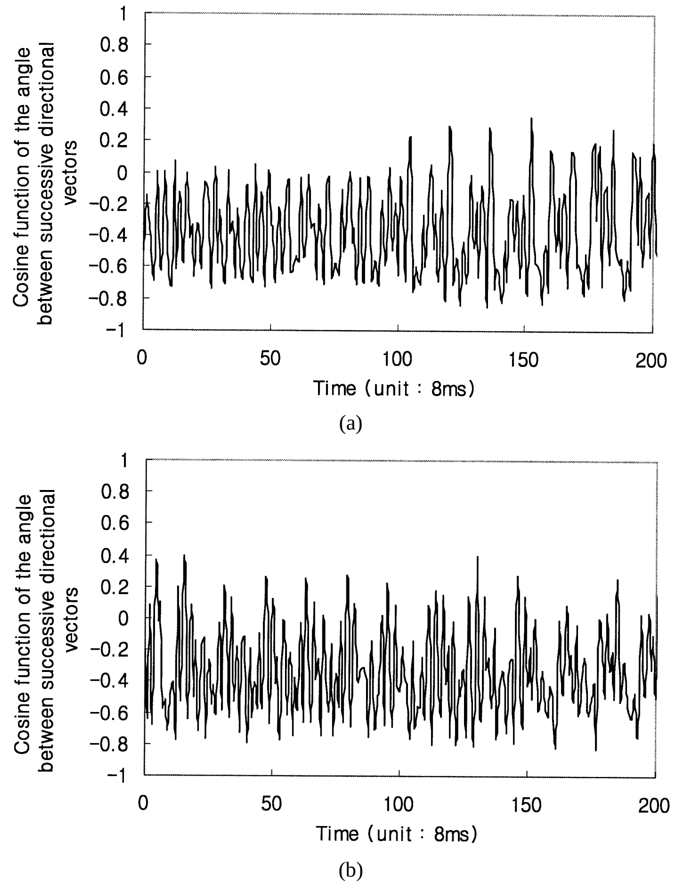

Fig. 4. The angle series over time from successive state vectors of the trajectory in the state space reconstructed with $d = 8$ and $T = 10$ for (a) a stationary EEG recording and (b) its surrogate data as a function of time.

tems, because very large amounts of noise can be lead to spurious, false negative results. The application of nonlinear noise-reduction methods prior to assessing smoothness might be useful in obtaining more reliable results.

From these considerations, we propose an empirical criterion for establishing determinism in practical applications: if S of a time series is near 0 or smaller than 0.3, then the time series is deterministic. If S is close to 1 or larger than 0.7, then we conclude that the time series is stochastic. The intermediate case, with $0.3 < S < 0.7$, is known to sometimes arise from deterministic time series. Since S is an informal measure, we suggest that a statistical analysis should compare the CTM of the original time series with those of their 20 surrogate data. In a later practical example, we also use a t-test to compare the CTM of an EEG record with the CTMs of 20 surrogate data sets.

B. Subjects and EEG Data Acquisition

Scalp EEGs were recorded in a conventional fashion from 20 normal subjects (9 male and 11 females) having a mean age of 31.3 ± 4.2 years. A 12-bit analog-to-digital converter digitized the signals from 20 monopolar electrodes that employed the International Federation 10–20 EEG reference system with a bimastoidean reference point. With the subjects in a relaxed state with eyes closed, EEG data (1 min, 15 000 data points) were recorded with a sampling frequency of 250 Hz. The data were high-pass filtered at 0.5 Hz and then low-pass filtered at 35 Hz. Recordings were made under an eyes-closed condition to obtain as many stationary epochs of EEG recording as possible. Potentials from 16 channels (F_7, T_3, Fp_1, F_3, C_3, P_3, O_1, F_8, T_4, T_5, T_6, Fp_2, F_4, C_4, P_4, and O_2) referenced against "linked earlobes" were amplified on a Nihon Kohden EEG-4421K recording unit using a time constant of 0.1s Each EEG record was judged by inspection to be free from electrooculographic and movement artifacts and to contain minimal electromyographic (EMG) activity.

C. Assessment of Stationarity

To assess the presence of determinism in stationary EEGs, we first selected stationary segments of the EEG record for analysis. The particular EEG epochs that would be used to assess determinism in the EEG record were selected using a criterion of the second-order weak stationarity, i.e., constant mean and variance with the autocorrelation depending only on the time difference.

We assessed the stationarity of time bins consisting of 250 data points (a duration of 1 s). First, the mean and variance for each bin were calculated, and then zones where these values did not change significantly for at least two consecutive bins were searched. Then whether the autocorrelation of the segment is dependent on time difference alone or not was checked. Next, we compared the statistical parameters of the total time series with those of the previously selected segment. If the differences between these statistical parameters were significant at a probability value <0.05, we discarded the corresponding segment. This procedure was followed to ensure that the selected segment represented the dynamics of the entire time series.

III. Results

Since one segment of stationary EEG was selected from each channel, 16 segments were obtained from each subject (the total number of stationary segments used: 320). The mean number of data points of the selected EEG segments from 16 electrodes of 20 subjects

TABLE III
THE COMPARISON OF THE CENTRAL TENDENCY MEASURES OF THE EEG DATA AT 16 ELECTRODES (CTM_{EEG}) AND THEIR SURROGATE DATA (CTM_{sur})

EEG data (n=20)	CTM_{EEG}	CTM_{sur}	S score	P
F_7	0.51±0.13	0.58±0.15	0.879±0.12	NS
F_3	0.48±0.14	0.53±0.14	0.906±0.13	NS
F_4	0.53±0.17	0.57±0.13	0.930±0.11	NS
F_8	0.47±0.14	0.56±0.15	0.839±0.12	NS
Fp_1	0.52±0.18	0.59±0.12	0.881±0.14	NS
Fp_2	0.58±0.19	0.64±0.17	0.906±0.12	NS
C_3	0.59±0.12	0.67±0.15	0.882±0.13	NS
C_4	0.52±0.14	0.54±0.13	0.963±0.14	NS
T_3	0.45±0.17	0.51±0.14	0.882±0.17	NS
T_4	0.56±0.19	0.59±0.12	0.949±0.11	NS
T_6	0.44±0.13	0.51±0.11	0.862±0.14	NS
P_3	0.53±0.12	0.57±0.16	0.939±0.15	NS
P_4	0.51±0.15	0.56±0.11	0.911±0.12	NS
O_1	0.41±0.17	0.51±0.13	0.804±0.18	NS
O_2	0.38±0.16	0.49±0.17	0.775±0.17	NS

was 1525 ± 245. In Fig. 3(a) and (b), we present a typical example of the mean and variance for each bin of the EEG record from a subject at F_3. The mean (and standard deviation) value for the entire EEG record in this case was 127.39 ± 2.63. The means and variances were stable from bin seven to bin nine satisfying the weak stationarity criteria. The autocorrelation of each segment as a function of only the time difference was confirmed.

Then the attractor for the stationary EEG segments was reconstructed using the embedding procedure. The smoothness of the attractor reconstructed from the short stationary EEG segments exhibited stable behavior and did not critically depend on embedding parameters for $10 < T < 30$ and $8 < d < 12$ for the EEG. Thus, T was chosen to be ten lags, and d was set to eight. Fig. 4(a) and (b) shows the angle variations of the successive tangent vectors of the trajectory reconstructed from a stationary EEG segment and of those reconstructed from its surrogate data. The angle variations for the stationary EEG were as irregular as those for the surrogate data. Both SODPs of the angle variations for the stationary EEG and the surrogate data exhibited large variability and were similarly distributed. As shown in Table III, the mean values of the CTMs for the stationary EEG and their 20 corresponding surrogate data for 20 subjects were not significantly different at each channel. The mean S values greater than 0.7 found in stationary EEGs indicates that the trajectories of the EEG in state space were not as smooth as those for stochastic time series having identical power spectra (Table III), suggesting no determinism within it.

IV. DISCUSSION

A method for identifying whether a short time series is deterministic or not is developed. We demonstrate that the method properly identifies the nature of several well-characterized dynamical systems and stochastic systems. When applied to a short sample of stationary EEG recordings, our results indicate that the stationary EEG record has minimal smoothness. These results suggest that scalp EEG recordings from normal subjects do not exhibit a low-dimensional deterministic structure. However, there is a possibility that large amounts of noise, either from measurement artifact or intrinsic noise sources within the brain, may have interfered with our ability to detect determinism in the EEG.

Our EEG findings are in agreement with several previous studies [10], [13], [14]. Blinowska and Malinowski have applied the Sugihara–May method to EEG recordings and reported that predictions using this method are similar to those using a linear autoregressive method [13]. Glass and his colleagues have employed the Kaplan–Glass method for deterministic dynamics for an EEG and found that real EEG record is not deterministic [14]. A more recent study performed by Jeong *et al.* [10] has examined determinism within the EEG by detecting the parallelness of nearby trajectories in state space reconstructed from the noise-reduced EEG after the use of a nonlinear noise reduction method. Compared with trajectories for its surrogate data, those for noise-reduced EEG data do not yield evidence for low-dimensional determinism.

Our novel method measures the smoothness of an attractor's trajectory. Thus, it is not applicable to map-type data such as interspike intervals of neuronal signals, R–R intervals of electrocardiograms, or other extremely-low-sampled data. The smoothness of the trajectory may depend on the sampling frequency of the data. However, in our experience, the difference between the CTMs of the original time series and its surrogate data is not critically sensitive to the sampling frequency, perhaps because the sampling frequency also affects the smoothness of the surrogate data. It is, nevertheless, worth noting that over- or under-sampling may lead to spurious results.

That the proposed method can be applied to short time series supports its usefulness for the analysis of physiological or experimental time series. It is, therefore, applicable to the analysis of time series such as the EEG, in which maintaining stationarity for a long duration is a difficult task.

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Postprocessing of 3-D Current Density Reconstruction Results With Equivalent Ellipsoids

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***Abstract*—A method of postprocessing and visualizing three–dimensional vector fields, such as current density reconstruction results, is presented. This method is based on equivalent ellipsoids fitted to the vector fields. The technique has been tested with simulated data and current density reconstructions based on bioelectromagnetic data obtained from a physical thorax phantom. Three different approaches based on: 1) longest distance; 2) dominant direction; and 3) principal component analysis, for fitting the equivalent ellipsoids are proposed. Multiple foci in vector fields are extracted by multiple ellipsoids which are fitted iteratively. The method enables statistical postprocessing for the sake of comparisons of different source reconstructions algorithms or comparisons of groups of patients or volunteers.**

***Index Terms*—Biomagnetics, biomedical electromagnetic imaging, inverse problems, statistics, visualization.**

I. INTRODUCTION

Current density reconstructions (CDRs) are used to assess cardiac activation [1], [2] and brain function [3], [4] based on noninvasively obtained magnetocardiogram (MCG), electrocardiogram (ECG), magnetoencephalogram (MEG), and electroencephalogram (EEG) data. CDRs are vector fields with each vector representing the current density in a volume element or on a surface element. Often only the magnitude maps of the CDRs are interpreted and represent the end point of analysis. However, a parameterization of the CDRs to facilitate statistical comparisons between data sets (from different individuals or from one individual in different conditions or times) would be desirable.

In a previous conference paper, we have introduced a parameterization technique for current density distributions which is based on equivalent ellipsoids and applied to two–dimensional problems [5]. In the current paper, we expand this technique to full three–dimensional (3-D) problems, present an improved algorithm for the estimation of the ellipsoids, compare a new principle component analysis (PCA) based

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