Biol. Cybern. 86, 335–342 (2002)
DOI 10.1007/s00422-001-0299-5

Biological Cybernetics

Detecting determinism in short time series, with an application to the analysis of a stationary EEG recording

Jaeseung Jeong[1], **John C. Gore**[2], **Bradley S. Peterson**[3]

[1] National Creative Research Initiative, Center for Neurodynamics and Department of Physics, Korea University, Seoul 136-701, South Korea
[2] Departments of Diagnostic Radiology and Applied Physics, Yale School of Medicine, Yale University, New Haven, CT 06520-8042, USA
[3] Department of Diagnostic Radiology and the Child Study Center, Yale School of Medicine, Yale University, New Haven, CT 06520-8042, USA

Received: 28 June 2001 / Accepted in revised form: 20 November 2001

Abstract. We have developed a new method for detecting determinism in a short time series and used this method to examine whether a stationary EEG is deterministic or stochastic. The method is based on the observation that the trajectory of a time series generated from a differentiable dynamical system behaves smoothly in an embedded phase space. The angles between two successive directional vectors in the trajectory reconstructed from a time series at a minimum embedding dimension were calculated as a function of time. We measured the irregularity of the angle variations obtained from the time series using second-order difference plots and central tendency measures, and compared these values with those from surrogate data. The ability of the proposed method to distinguish between chaotic and stochastic dynamics is demonstrated through a number of simulated time series, including data from Lorenz, Rössler, and Van der Pol attractors, high-dimensional equations, and 1/f noise. We then applied this method to the analysis of stationary segments of EEG recordings consisting of 750 data points (6-s segments) from five normal subjects. The stationary EEG segments were not found to exhibit deterministic components. This method can be used to analyze determinism in short time series, such as those from physiological recordings, that can be modeled using differentiable dynamical processes.

1 Introduction

An important problem in the study of aperiodic and apparently irregular time series is determining whether the time series arises from a stochastic process or whether it has deterministic components that are generated from chaotic dynamics having finite degrees of freedom. Whether a time series has deterministic components or not in turn dictates what approaches are appropriate for investigating the time series and its generating system. If the time series is a stochastic signal, then we can examine it using statistical methods, such as estimating its mean, variance, or autocorrelation. In contrast, if the time series has deterministic components, we can investigate and quantify its properties using dynamical methods and then model its generating system using differential equations or difference maps. Conventional spectral analysis is unable to distinguish deterministic chaos from random processes, because both types of time series may have continuous broadband power spectra.

Several methods of nonlinear dynamical analysis have been developed previously to detect determinism in a time series (Sugihara and May 1990; Kaplan and Glass 1992, 1993; Kennel and Isabelle 1992; Theiler et al. 1992; Tsonis and Elsner 1992; Wayland et al. 1993; Salvino and Cawley 1994). These methods are all based on the assumption that a trajectory in the phase space reconstructed from a deterministic time series behaves similarly to nearby trajectories as time evolves. In addition, each of these methods measures the predictability or continuity of the trajectories over all regions of the attractor, in order to provide a global measure of predictability in the system. Thus, a large number of data points are required to have sufficient information on nearby trajectories either to compare their future behaviors or to construct the entire attractor. Furthermore, if the time series under study is nonstationary, the application of these methods can lead to spurious results. Acquiring this large number of data points from a stationary time series is almost impossible when working with real biological systems. Newer, more practical methods are needed for assessing determinism in shorter and more realistic time series.

Existing methods for assessing determinism have been applied repeatedly, for example, to the analysis of EEG recordings (Blinowska and Malinowski 1991; Mees 1992; Glass et al. 1993; Jeong et al. 1999), but it has been difficult to conclude whether those recordings are deterministic. The difficulty arises in part because it has been difficult to obtain sufficiently long, continuous

Correspondence to: J. Jeong
(e-mail: jsjeong@complex.korea.ac.kr,
Tel.: +82-2-32904288, Fax: +82-2-32903534)

EEG recordings. Long recordings tend to become non-stationary – their mean, variance, and probability distributions tend to change with time. Statistical tests of the stationarity of EEG recordings have yielded variable estimates for the duration during which the EEG record is stationary, ranging from several seconds to several minutes (McEwen and Anderson 1975; Sugimoto et al. 1977; Blanco et al. 1995).

The aim of this study was to develop a method for detecting determinism in short time series, such as physiological recordings, and then apply it to stationary segments of an EEG record. The method is based on the observation that the trajectory of an attractor evolves smoothly in phase space when the time series from which it has been reconstructed has been generated by a differentiable dynamical system. We therefore measure the angles of successive directional vectors of attractor trajectories in phase space and examine their variability. We compare the irregularity of the variations in trajectory of the original time series with those of its surrogate data. This method does not require an excessively large number of data points because it does not need to construct the entire attractor to measure the variability of its component trajectories. We then apply this method to several simulated time series – including Rössler, Lorenz, and Van der Pol data, as well as a high-dimensional signal and autocorrelated noise – to test its ability to distinguish between chaotic and stochastic behaviors. Finally, we apply the method to the analysis of real physiological data from short, stationary segments of an EEG recording.

2 Methods

2.1 Algorithm for detecting determinism

We first transform a one-dimensional time series into a multidimensional phase space. The reconstruction of the attractor in the phase space is carried out through the technique of plotting delay coordinates (Takens 1981; Eckmann and Ruelle 1985). Let an observed time series $x(t)$ be the output of a differentiable dynamical system f^t on an m-dimensional manifold M. In order to unfold the projection back onto a multivariate phase space that is a representation of the original system, we use the delay coordinates

$$\mathbf{X}(t) = [x(t), x(t+T), \ldots, x(t+(d-1)T)] \quad (1)$$

from a single time series $x(t)$ after performing an embedding procedure. $X(t)$ is one point of the trajectory in the phase space at time t, $x(t + iT)$ are the coordinates in the phase space corresponding to the time-delayed values of the time series, T is the time delay between the points of the time series considered, and d is the embedding dimension.

The choice of an appropriate time delay T and embedding dimension d are important for the success of reconstructing the attractor using a finite number of data points. For the time delay T, we use the first local minimum of the average mutual information between the set of measurements $x(t)$ and $x(t + T)$ (Fraser and Swinney 1986).

We use the minimum (optimal) embedding dimension in the reconstruction procedure. The algorithm for estimating the minimum embedding dimension is based on the idea that in the passage from dimension d to dimension $d + 1$, one can differentiate between points on the orbit that are true and those that are false neighbors. A false neighbor is a point in the data set that is a neighbor solely because we are viewing the orbit (the attractor) in too small an embedding space ($d < d_{\min}$). When we have achieved a large-enough embedding space ($d \geq d_{\min}$), all neighbors of every orbit point situated within the multivariate phase space will be true neighbors. The detailed algorithm is presented in Jeong et al. (1998a).

Our test for determinism in a time series is based upon assessing the smoothness of a trajectory in phase space of the attractor that has been reconstructed from the time series. The trajectory of its attractor should evolve smoothly because the differentiability of the original system is preserved in phase space (Takens 1981). Thus, the angles between successive directional vectors that are tangential to the trajectory should vary smoothly, whereas the directions of directional vectors reconstructed from noise should be random, even if the noise is autocorrelated. Figure 1a shows the two-dimensional phase-space plot $x(t)$ versus $x(t + 1)$ for the x-component of the Lorenz equations $[\dot{x} = 10(y - x), \dot{y} = 28x - y - xz, \dot{z} = -\frac{8}{3}z + xy, x = 10(y - x)]$ with $t = 0.01$. Figure 1b shows the plot for a random signal with an identical power spectrum. The number of data points is 2000, the embedding dimension is 7, and the time delay is 15. Although both time series have the same autocorrelation function, the trajectory of the Lorenz attractor is very smooth, while that for random noise is highly irregular.

We next measure the angles between successive directional vectors iteratively along the trajectory in phase space. We define the directional vector of a trajectory in the phase space as follows:

$$\mathbf{Y}(t) = \mathbf{X}(t+1) - \mathbf{X}(t) \quad (2)$$

Then the angles between successive directional vectors are computed by

$$R(t) = \frac{\mathbf{Y}(t+1) \cdot \mathbf{Y}(t)}{|\mathbf{Y}(t+1)||\mathbf{Y}(t)|} \quad (3)$$

$R(t)$ is the cosine function of the angle between successive directional vectors. If successive directional vectors in the trajectory are parallel, then R is 1. The value of R decreases as the angle between successive directional vectors increases.

Although the absolute value of the angles between directional vectors may depend on the correlation of the time series, the regularity of the angles reflects the determinism of the time series. Figure 2 demonstrates, for example, that the cosine function of the angles between successive directional vectors for the Lorenz attractor behaves more smoothly than that for random noise time series that has an identical power spectrum.

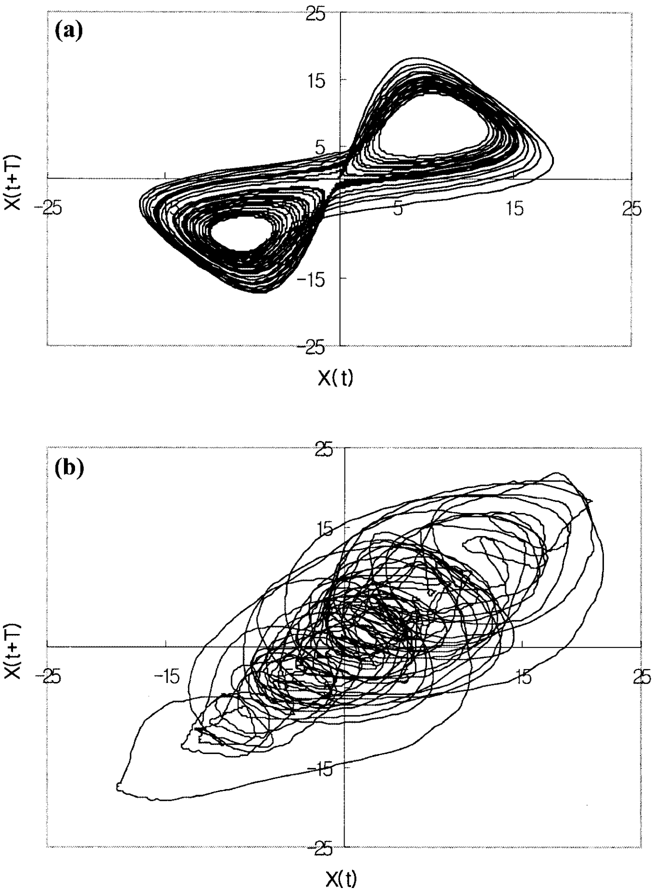

Fig. 1a,b. Two-dimensional phase portraits for the x-component of Lorenz equations (**a**) and for a Gaussian random noise having an identical power spectrum (**b**). The number of data points is 2000 and the time delay T is 15

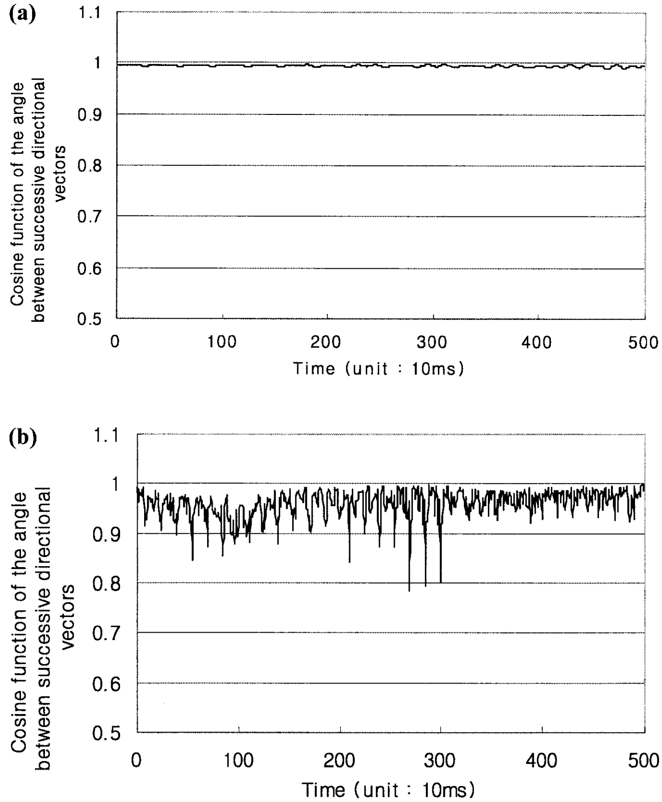

Fig. 2a,b. The angle series of the successive directional vectors of the trajectory in the phase space reconstructed from the Lorenz data (2000 data points) (**a**) and those from its surrogate data (**b**) as functions of time with embedding dimension of 7 and time delay of 15

Next, we use a second-order difference plot (SODP) and a central tendency measure (CTM) to quantify the irregularity, or angle variations, of the successive directional vectors along the trajectory. The SODP is a graphical representation of the rate of variability: $(R_{n+2} - R_{n+1})$ versus $(R_{n+1} - R_n)$, as proposed by Cohen et al. (1996). Maurer et al. (1997) showed that the SODP is efficient for quantifying the irregularity of the time series graphically. The CTM provides a rapid quantitative estimate for the variability in the SODP. The CTM is computed by the average length of the points from the origin in the SODP:

$$\mathrm{CTM} = \frac{1}{N-2}\sum_{n=1}^{N-2}\sqrt{(R_{n+2}-R_{n+1})^2 + (R_{n+1}-R_n)^2} \quad (4)$$

where N is the total number of points. A larger value for the CTM of the angle variations indicates a less smooth attractor trajectory. We define the index of smoothness of the trajectory S as a ratio of the CTM of the angle variations of the successive directional vectors for the original time series to that for its surrogate data. It is an effective measure for smoothness of the trajectory and for determinism in the time series.

We use the method of surrogate data generation to help detect nonlinear determinism. Surrogate data are linear stochastic time series that have the same power spectra as the raw time series. They are randomized to ensure the absence of any deterministic nonlinear structure. We use "iteratively refined surrogate data," which have the same autocorrelation function, Fourier power spectrum, and probability distribution as the original time series $x(t)$ (Theiler et al. 1992; Schreiber and Schmitz 2000). We use the end-point mismatch measure d_{jump} and the mismatch in the first derivative, as proposed by Schreiber and Schmitz (2000), to minimize the periodicity artifacts (Ehlers et al. 1998) by selecting a subinterval of the time series such that the end points match as closely as possible.

Figure 3 presents the SODPs of the angles between successive directional vectors for the Lorenz attractor and its surrogate data. The SODP for the Lorenz attractor with low variability had points clustered around the origin, whereas the SODP for its surrogate data had a much wider distribution of points, indicating high variability of successive directional vectors. The CTM values for the Lorenz attractor and its surrogate data were 0.002 and 0.035, respectively. We generated 19 surrogate data of the Lorenz attractor to test the stability of the CTM values of the surrogate data. The average CTM value and standard deviation of 19 surrogate data sets from the Lorenz attractor was 0.034 ± 0.003, indicating that the

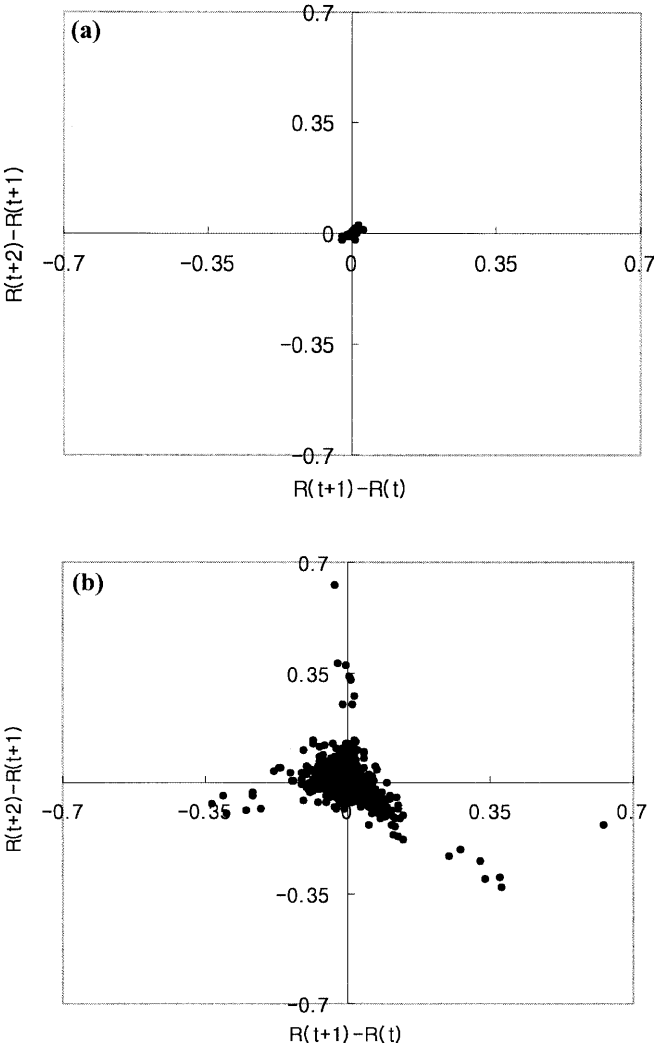

Fig. 3a,b. Second-order difference plots of the angle series of the successive directional vectors of the trajectory reconstructed from the Lorenz data (**a**) and its surrogate data (**b**)

CTM is nearly constant for all of the surrogate data sets. Student's *t*-tests showed that the CTM value for the Lorenz attractor was significantly different from those for its surrogate data sets ($P < 0.0001$). The smoothness of the trajectory S for the Lorenz attractor (i.e., the ratio of the CTM for the original Lorenz attractor to that for its surrogate data) is 0.057, indicating that the Lorenz attractor is very smooth. We also applied this method to the Rössler data, Van der Pol data, the high-dimensional coupled equations, and autocorrelated noise (Table 1).

To examine the applicability of this method to high-dimensional systems, we estimated the smoothness of the high-dimensional time series. The data were generated from nonlinear coupled equations [$\dot{x}_1 = \dot{x}_2$, $\dot{x}_2 = \frac{(x_5-25)}{3}\sin 30t + 3x_7 \sin 65t + x_{11} \quad \sin 80t - 3|x_6|x_2 - x_9x_1$] of 12 variables including the Lorenz equation [$\dot{x}_3 = 10(x_4 - x_3), \dot{x}_4 = -x_3x_5 + 28x_3 - x_4, \dot{x}_5 = x_3x_4 - \frac{8}{3}x$], the Ueda equation [$\dot{x}_6 = x_7, \dot{x}_7 = -0.1x_7 - x_6^3 + 12\cos t$], the two-well potential Duffing–Homes equation [$\dot{x}_8 = x_9, \dot{x}_9 = 0.15x_9 + 0.5x_8(1 - x_8^2) + 0.15\cos 0.8t$], and the Rössler equation [$\dot{x}_{10} = -(x_{11} + x_{12}), \dot{x}_{11} = x_{10} + 0.15x_{11}, \dot{x}_{12} = 0.15 + x_{12}(x_{10} - 10)$]. The high-dimensional signal was found to have smoother trajectories than its surrogate data (Table 1), demonstrating the relevance of this method for the assessment of determinism in high-dimensional dynamical systems.

We also tested the ability of the method to avoid the false-positive designation of determinism for autocorrelated noise. We used a sequence of numbers that was generated by tossing imaginary dice successively and that produces 1/f noise, as shown by Gardner (1992). We used 40 dice, each with 12 sides. We first threw all 40 dice and calculated their sum. Next, we randomly selected and threw again only 15 of the dice but recalculated the sum of all 40 dice. The resulting sequence of sums is autocorrelated, with a power spectrum having a 1/f distribution in the range of about 10–30 Hz. The detailed algorithm is presented in Jeong et al. (1998c). The SODP and CTM for 2000 data points in this autocorrelated noise time series and for its surrogate data were estimated using an embedding dimension of 7 and a time delay of 15. Both the original and surrogate time series exhibit a large variability in their SODPs. The surrogate data had a somewhat higher SODP variability, but the CTMs for the 1/f noise and its surrogate data did not differ significantly (Table 1).

From these considerations, we propose an empirical criterion for establishing determinism in practical applications: if S of a time series is near 0 or smaller than 0.3, then the time series is deterministic; whereas if S is close to 1 or larger than 0.5, then we conclude that the time series is stochastic. The intermediate case, with $0.3 < S < 0.5$, is known to sometimes arise from deterministic time series. Since S is, however, an informal measure, we suggest that in this case a statistical analysis should compare the CTM of the original time series with those of its surrogate data. In a later practical example, for instance, we used a *t*-test to compare the CTM of an EEG record with the CTMs of 19 surrogate data sets.

The values of R and CTM are closely related to the autocorrelation function evaluated at a lag of 1, indicating that CTM may not be optimally powerful for a test of nonlinearity. Thus, entropy can also be used as an alternative nonlinear measure of the irregularity of the

Table 1. The central tendency measures of the original time series (CTM_{sig}) and their surrogate data (CTM_{sur}) from different dynamical systems (NS, not significant)

Data	CTM_{sig}	CTM_{sur}	S	P
Lorenz data	0.002	0.034 ± 0.003	0.057	<0.0001
Rössler data	0.001	0.35 ± 0.09	0.030	<0.0001
Van der Pol data	0.0002	0.68 ± 0.08	0.0003	<0.0001
High-dimensional signal	0.0088	0.54 ± 0.11	0.016	<0.0001
Autocorrelated noise	0.119	0.133 ± 0.068	0.895	NS

angle variations of successive directional vectors of the attractor reconstructed from a time series (Eckmann and Ruelle 1985). To estimate the entropy of the angle variations, the angle variations are first converted to probability values by defining $p(n) = P(n)/\sum P(n)$. An entropy function (in bits) can then be estimated as $S = -\sum_n p(n)\log_2[p(n)]$.

2.2 Subjects and EEG data acquisition

Scalp EEGs were recorded in a conventional fashion from five normal subjects (three males and two females having a mean (± SD) age of 28.3 ± 3.8 years. A 12-bit analog-to-digital converter digitized the signals from 20 monopolar electrodes that employed the international 10–20 EEG reference system with a bimastoidean reference point. With the subjects in a relaxed state with eyes closed, 30 s of data (3750 data points) were recorded at a sampling frequency of 125 Hz. The data were high-pass filtered at 0.5 Hz and then low-pass filtered at 35 Hz. Recordings were made under an eyes-closed condition in order to obtain as many stationary epochs of EEG recording as possible. Potentials from 16 channels (F7, T3, Fp1, F3, C3, P3, O1, F8, T4, T5, T6, Fp2, F4, C4, P4, and O2) referenced against "linked earlobes" were amplified on a Nihon Kohden EEG-4421K recording unit using a time constant of 0.1 s. Each EEG record was judged by inspection to be free from electrooculographic and movement artifacts, and containing minimal electromyographic activity.

2.3 Assessment of stationarity

In order to assess the presence of determinism in the EEG, we first had to select stationary segments of the EEG record for analysis. If the probability distribution $f(x)$ of a time series is normal or Gaussian, it can be completely characterized by its mean and variance. Thus, if we have a normal distribution, the existence of a fixed mean and variance should be enough to ensure the stationarity of a Gaussian process. A less restrictive requirement, called *weak stationarity* of order n, is that the moments up to order n depend only on time differences (Jenkins and Watts 1968). In this study, we used second-order stationarity ($n = 2$) and a demonstration of normality to ensure stationarity. We used the following procedure, following Blanco et al. (1995), to ensure stationarity of the particular EEG epochs that would be used to assess determinism in the EEG record.

We assessed the stationarity of time bins consisting of 250 data points (a total duration of 2 s). First, we calculated the mean and variance for each bin and looked for zones where these values did not change significantly for at least two consecutive bins. Then we constructed the corresponding histogram for this EEG segment, and used the Kolmogorov–Smirnov test to verify that the data were normally distributed. We then compared the statistical parameters of the total time series with those of the previously selected segment. If the differences between these statistical parameters were significant at a probability value of less than 0.05, then we discarded the corresponding segment. This procedure was followed to ensure that the selected segment represented the dynamics of the entire time series.

3 Results

In Fig. 4 we show the mean and variance for each bin of the EEG data from a single subject at F3. The mean and standard deviation of the EEG potential for the entire EEG record was 127.39 ± 2.63. The means and variances were stable from bin 7 to bin 9, satisfying the weak stationarity criteria. The histograms for the EEG potentials were normally distributed (Kolmogorov–Smirnov $P < 0.05$), as shown in Fig. 5.

We reconstructed the attractor for the stationary EEG segments using the embedding procedure. The time delays were chosen to be 12–18 lags from the first local minimum of the average mutual information between the set of the EEG $x(t)$ and $x(t + T)$. The minimum embedding dimensions were therefore selected as 14–17 for the stationary EEG segments.

Figure 6 shows the angle variations of the successive directional vectors of the trajectory reconstructed from a

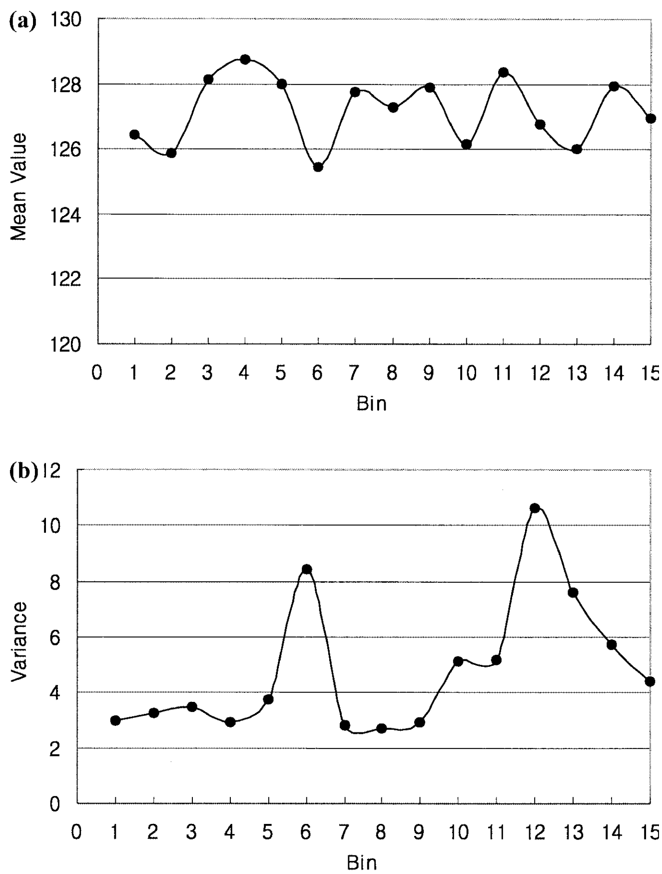

Fig. 4a,b. Stationarity of the EEG: **a** mean values for bins of 125 data points for an EEG time series at F3 from a single subject; **b** variance for bins of 125 data points for an EEG time series from the same subject

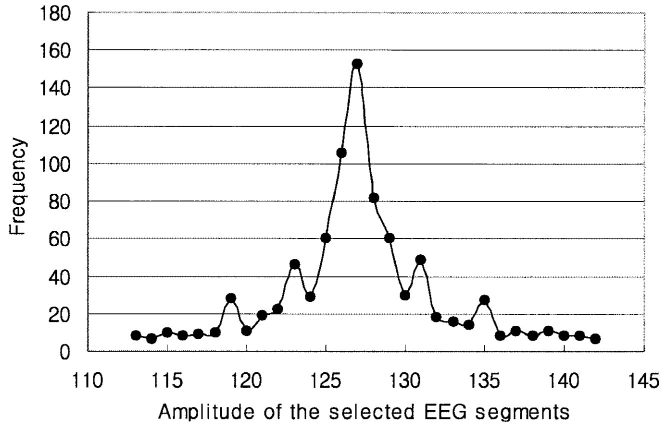

Fig. 5. Histogram for the time series from selected EEG epochs

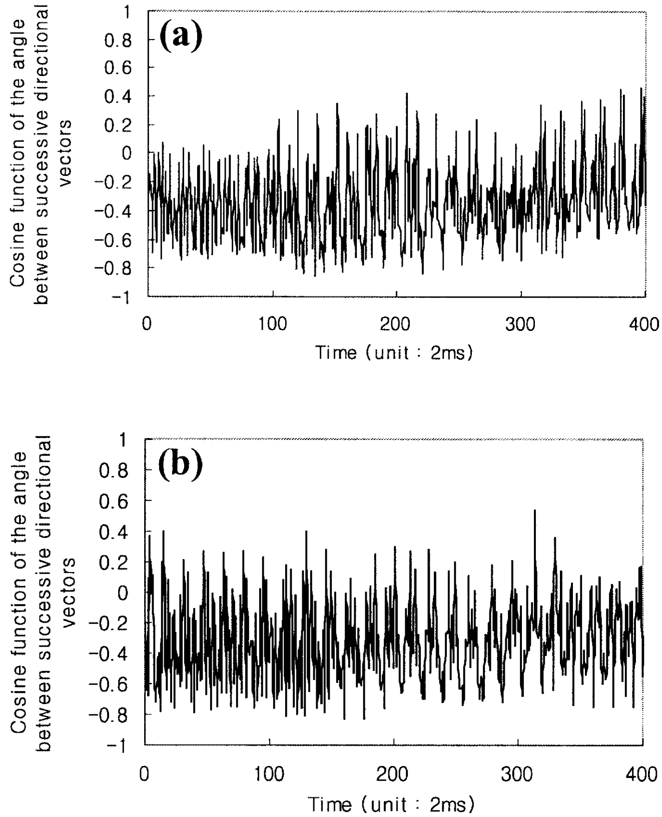

Fig. 6a,b. The angle series of the successive state vectors of the trajectory in the phase space reconstructed from the stationary EEG data (**a**) and its surrogate data (**b**), as functions of time with embedding dimension of 14 and time delay of 12

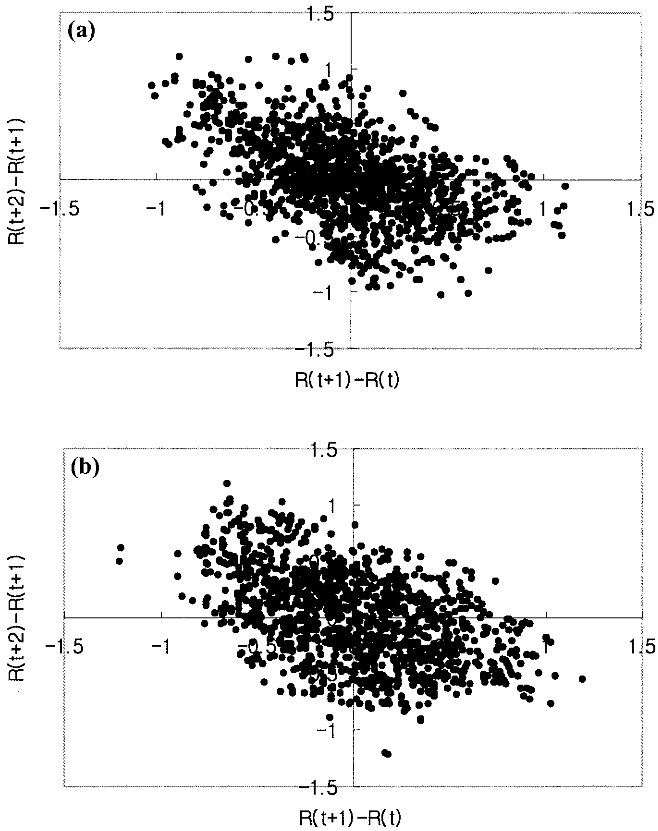

Fig. 7a,b. Second-order difference plots of the angle series from the stationary EEG data (**a**) and its surrogate data (**b**)

stationary EEG at F3 from a single subject and those reconstructed from its surrogate data. The angle variations for the stationary EEG were as irregular as those for the surrogate data. Figure 7 shows the SODPs of the angle variations for the stationary EEG and its surrogate data. Both SODPs exhibited large variability and were similarly distributed. The mean values of the CTMs for the stationary EEG (0.52 ± 0.14) and its surrogate data (0.58 ± 0.16) for five subjects were not significantly different ($P = 0.102$). The mean S of the stationary EEGs was found to be 0.89 ± 0.11, indicating that the trajectories of the EEGs in phase space were not as smooth as those for stochastic time series having identical power spectra. Additionally, the entropy of the angle variations for the stationary EEG (5.43 ± 2.35 bits/s) and its surrogate data (5.83 ± 2.65 bits/s) were not significantly different.

It is possible that our inability to detect determinism in the EEG record was due to the presence of noise in the recording of a high-dimensional dynamical system. To assess whether noise produces false negatives using analyses of smoothness, we applied the method to the assessment of a high-dimensional deterministic signal containing uniformly distributed noise. The signal contained 50% white noise and nevertheless had a CTM of 0.098, significantly smaller than the CTMs of its surrogate data (0.139 ± 0.038; $P < 0.01$). However, statistically significant differences were not found between the CTM of the signal containing 100% noise (0.196) and the CTMs of its surrogate data (0.213 ± 0.096), indicating that very large amounts of noise can lead to spurious false-negative results. Although we did not detect any determinism in the stationary EEG using our method, we cannot exclude the possibility that large amounts of noise – either from measurement artifact or intrinsic noise sources with the brain – may have interfered with our ability to detect determinism in the EEG.

4 Discussion

We have developed a method for identifying whether short segments of a time series are deterministic. We have verified that the method properly identifies the nature of several well-characterized dynamical systems and stochastic processes. When applied to a short sample of a stationary EEG waveform, our results indicate that the stationary EEG record has minimal smoothness. These results suggest that the EEG record is not deterministic.

Our EEG findings are in agreement with several previous studies (Blinowska and Malinowski 1991; Glass et al. 1993; Jeong et al. 1999). Blinowska and Malinowski (1991) applied the Sugihara–May method to EEG recordings and reported that predictions using this method were similar to those using a linear autoregressive method. Glass et al. (1993) employed the Kaplan–Glass method for deterministic dynamics for both a real EEG and a simulated EEG generated by a neural network model. They found similar orientations of tangents to the trajectory in a given small region of phase space from simulated EEGs, but not from real EEGs. Thus they concluded that real EEG record was not deterministic. A more recent study performed by Jeong et al. (1999) examined determinism in the EEG by detecting the smoothness of trajectories in phase space reconstructed from the noise-reduced EEG using a nonlinear noise-reduction method. Compared with trajectories for its surrogate data, those for noise-reduced EEG data did not yield evidence for low-dimensional determinism.

Cerf et al. (1997, 1999) found the presence of low-dimensional neuronal dynamics in short episodes of EEG α-waves with a duration of 6 s using correlation integrals, whereas no low-dimensional dynamics was found in EEG α-waves with durations of 10 s or more. The duration of 6 s is the upper duration for which deterministic dynamics has been found in the form of low-dimensional α-episodes (Cerf et al. 1997, 1999). We also used stationary EEG recordings with a duration of 6 s satisfying the weak stationarity criteria. However, we could not detect any deterministic structure in the stationary EEG data. One of the possible reasons why we did not detect any deterministic structure in the short EEG epochs is that the α-episodes in the EEG recordings analyzed were rather scarce. The probability of α-episodes is usually very low.

Despite the absence of compelling evidence for determinism in the EEG, nonlinear dynamical analyses of the EEG in which its correlation dimension and first positive Lyapunov exponent are estimated have proven to be useful in differentiating normal and pathological brain states (Babloyantz et al. 1985; Babloyantz and Destexhe 1986; Frank et al. 1990; Jeong et al. 1998b; Lehnertz and Elger 1998). However, caution should be used in interpreting these particular nonlinear dynamical measures, which are most appropriately regarded as operationally defined measures of complexity. They may not be appropriate measures for differentiating between periodic, chaotic, or stochastic dynamics in the formal sense.

We have proposed a new method for the assessment of determinism that is based on the smoothness of the trajectory in phase space of a short time series. The validity of the assumption that is central to our approach – that the first differentiability of the attractor's trajectory is useful in the detection of determinism – was mathematically proved by Ortega and Louis (1998) using a measure-based approach. They showed that statistical differentiability of the natural invariant measure along the reconstructed trajectory implies smoothness or determinism in a time series.

Because this method measures the smoothness of an attractor's trajectory, it is not applicable to map-type data such as interspike intervals of neuronal signals, R-R intervals of electrocardiograms, or other extremelylow-sampled data. One of the problems with most methods for detecting determinism is the inefficiency of the methods in the presence of noise. The Sugihara–May method cannot distinguish correlated noise from a chaotic time series (Cazellus and Ferrare 1992; Ensminger and Vinson 1994). These authors also showed that such techniques are vulnerable to producing false-negative conclusions in the presence of noise. The smoothness of an attractor's trajectory in phase space may also be corrupted by the presence of noise. Thus, the application of nonlinear noise-reduction methods prior to assessing smoothness may be useful in obtaining the most reliable results. However, if an algorithm to reduce noise in a time series operates by smoothing the trajectory in phase space, e.g., as Sauer's method does (Sauer 1992), then it may also affect the conclusions regarding determinism in the time series.

That the proposed method can be applied to short time series supports its usefulness for the analysis of physiological and other experimental time series. It is therefore applicable to the analysis of time series such as the EEG, in which maintaining stationarity for a long duration proves difficult.

Acknowledgements. The authors thank Jae Hoon Lee at St. Mary's Hospital in Taejon for experimental help during this study. We also appreciate the valuable comments of an anonymous reviewer. This work was supported by the Creative Research Initiatives of the Korean Ministry of Science and Technology, and by grant MH01232 from the National Institutes of Health, Bethesda, Md., and by a grant from the Tourette Syndrome Association.

References

Babloyantz A, Destexhe A (1986) Low-dimensional chaos in an instance of epilepsy. Proc Natl Acad Sci USA 83: 3515–3517

Babloyantz A, Salazar JM, Nicolls C (1985) Evidence of chaotic dynamics of brain activity during the sleep cycle. Phys Lett A 111: 152–156

Blanco S, Garcia H, Quian Quiroga R, Romanelli L, Rosso OA (1995) Stationarity of the EEG series. IEEE Eng Med Biol 14: 395–399

Blinowska KJ, Malinowski M (1991) Nonlinear and linear forecasting of the EEG time series. Biol Cybern 66: 159–165

Cazellus B, Ferrare B (1992) How predictable is chaos? Nature 355: 25–26

Cerf R, Daoudi A, Ould Henoune M, el Ouasdad EH (1997) Episodes of low-dimensional self-organized dynamics from electroencephalographic α-signals. Biol Cybern 77: 235–245

Cerf R, el Ouasdad EH, el Amri M (1999) EEG-detected episodes of low-dimensional self-organized cortical activity and the concept of a brain attractor. In: Uhl C (ed) Analysis of neurophysiological brain functioning. Springer, Berlin Heidelberg New York, pp 126–144

Cohen ME, Hudson DL, Deedwania PC (1996) Applying continuous chaotic modeling to cardiac signal analysis. IEEE Eng Med Biol 15: 97–102

Eckmann JP, Ruelle D (1985) Ergodic theory of chaos and strange attractors. Rev Mod Phys 57: 617–656

Ehlers CL, Havstad J, Prichard D, Theiler J (1998) Low doses of ethanol reduce evidence for nonlinear structure in brain activity. J Neurosci 18: 7474–7486

Ensminger PA, Vinson M (1994) Deterministic non-linear dynamics in Phycomyces sporangiophores. J Theor Biol 170: 259–266

Frank GW, Lookman T, Nerenberg MAH, Essex C (1990) Chaotic time series analysis of epileptic seizures. Physica D 46: 427–438

Fraser AM, Swinney HL (1986) Independent coordinates for strange attractors from mutual information. Phys Rev A 33: 1134–1140

Gardner M (1992) Fractal music hypercards and more. Freeman, New York

Glass L, Kaplan DT, Lewis JE (1993) Tests for deterministic dynamics in real and model neural networks. In: Jansen BH, Brandt ME (eds) Proceedings of the Second Annual Conference on Nonlinear Dynamical analysis of the EEG. World Scientific, Singapore, pp 233–240

Jenkins GM, Watts D (1968) Spectral analysis and its application. Holden-Day, San Francisco

Jeong J, Kim DK, Chae J-H, Kim SY, Ko H-J, Paik IH (1998a) Nonlinear analysis of the EEG of schizophrenics with optimal embedding dimension. Med Eng Phys 20: 669–676

Jeong J, Kim SY, Han SH (1998b) Nonlinear analysis of chaotic dynamics underlying EEGs in patients with Alzheimer's disease. Electroencephalogr Clin Neurophys 106: 220–228

Jeong J, Joung MK, Kim SY (1998c) Quantification of emotion by nonlinear analysis of the chaotic dynamics of electroencephalograms during perception of 1/f music. Biol Cybern 78: 217–225

Jeong J, Kim MS, Kim SY (1999) Test for low-dimensional determinism in electroencephalograms. Phys Rev E 60: 831–837

Kaplan DT, Glass L (1992) A direct test for determinism in a time series. Phys Rev Lett 68: 427–430

Kaplan DT, Glass L (1993) Coarse-grained embeddings of time series: random walks, Gaussian random processes, and deterministic chaos. Physica D 64: 431–454

Kennel MB, Isabelle S (1992) Method to distinguish possible chaos from colored noise and to determine embedding parameters. Phys Rev A 46: 3111–3118

Kennel MB, Brown R, Abarbanel HDI (1992) Determining embedding dimension for phase-space reconstruction using a geometrical construction. Phys Rev A 45: 3403–3411

Lehnertz K, Elger C (1998) Can epileptic seizures be predicted? Evidence from nonlinear time series analysis of brain electrical activity. Phys Rev Lett 80: 5019–5022

Maurer P, Wang H-D, Babloyantz A (1997) Time structure of chaotic attractors: a graphical view. Phys Rev E 56: 1188–1196

McEwen JA, Anderson GB (1975) Modeling the stationarity and Gaussianity of spontaneous electroencephalographic activity. IEEE Trans Biomed Eng 22: 361–369

Mees AI (1992) Tesselations and dynamical systems. In: Casdagli M, Eubank S (eds) Nonlinear modeling and forecasting. Addison-Wesley, New York, pp 3–12

Ortega GJ, Louis E (1998) Smoothness implies determinism in time series: a measure based approach. Phys Rev Lett 81: 4345–4348

Salvino LW, Cawley R (1994) Smoothness implies determinism: a method to detect it in time series. Phys Rev Lett 73: 1091–1094

Sauer T (1992) A noise reduction method for signals from nonlinear systems. Physica D 58: 193–201

Schreiber T, Schmitz A (2000) Surrogate time series. Physica D 142: 346–382

Sugihara G, May RM (1990) Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. Nature 344: 734–741

Sugimoto H, Ishii N, Iwata A, Suzumura N (1977) Stationarity and normality test for biomedical data. Comput Programs Biomed 7: 293–304

Takens F (1981) Detecting strange attractors in turbulences. In: Rand D, Young L-S (eds) Dynamical systems and turbulence. Springer, Berlin Heidelberg New York, pp 366–381

Theiler J, Eubank S, Longtin A, Galdrikian B, Farmer JD (1992) Testing for nonlinearity in time series: the method of surrogate data. Physica D 58: 77–94

Tsonis AA, Elsner JB (1992) Nonlinear prediction as a way of distinguishing chaos from random fractal sequences. Nature 358: 217–220

Wayland R, Broomley D, Pickett D, Passamante A (1993) Recognizing determinism in a time series. Phys Rev Lett 70: 580–582